Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SOME PROBLEMS OF QUADRATIC RATIONAL DIFFERENCE EQUATIONS[1]

AIJA AŅISIMOVA[1] and INESE BULA[1,2]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: aija-anisimova@inbox.lv, ibula@lanet.lv

Difference equations have been increasingly used as mathematical models in many disciplines including genetics, epidemiology, ecology, physiology, neural networks, psychology, engineering, physics, chemistry and social sciences.

We consider the second-order quadratic rational difference equation

$$x_{n+1} = \frac{\alpha + \beta x_n x_{n-1} + \gamma x_{n-1}}{A + B x_n x_{n-1} + C x_{n-1}},\ n = 0, 1, 2, \ldots \tag{1}$$

with nonnegative parameters and with arbitrary nonnegative initial conditions such that the deminator is always positive. These equations are research in papers [1; 2]. The authors are investigate the global stability character, the periodic nature, and the boundedness of solutions of a rational difference equation (1), and they give several open problems and conjuctures about these equations. These results offer for the development of the basic theory of the global behavior of solutions of nonlinear defference equations.

In our talk we discuss about difference equation

$$x_{n+1} = \beta + \frac{1}{x_n x_{n-1}},\ n = 0, 1, 2, \ldots . \tag{2}$$

We would like to prove that every positive solution of equation (2) has a finite limit. Our results continue the development of the basic theory of the global behavior of solutions of nonlinear difference equations.

## REFERENCES

[1] A.M. Amleh, E. Camouzis and G. Ladas. On the Dynamics of a Rational Difference Equation, Part 1. *International Journal of Difference Equations*, **3** (1):1–35, 2008.

[2] A.M. Amleh, E. Camouzis and G. Ladas. On the Dynamics of a Rational Difference Equation, Part 2. *International Journal of Difference Equations*, **3** (2):195–225, 2008.

[1] This work was partially supported by the Latvian Council of Science research project 09.1220

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# AN OPTIMAL CONTROL OF PROBABILITY DENSITY FUNCTIONS OF ONE-DIMENSIONAL STOCHASTIC PROCESSES[1]

MARIO ANNUNZIATO

*Dipartimento di Matematica - Università degli Studi di Salerno*

Via Ponte Don Melillo, 84084 Fisciano (SA)

E-mail: mannunzi@unisa.it

An optimal control strategy of stochastic processes is formulatedby using the one-dimensional Fokker-Planck equation.The definition of the control objectives are based on the probability density functions,and the optimal control is achieved as the minimizer of the objective under the constraint given by the Fokker-Planck equation.The purpose of the control is matching a final target configuration, or tracking a desired trajectory by means of a receding-horizon algorithm over a sequence of time windows.

**REFERENCES**

[1] M. Annunziato and A. Borzì. Optimal control of probability density functions of stochastic processes. *Mathematical Modelling and Analysis*, **15** (8):393–407, 2010.

[1]The author acknowledges financial support from *Istituto Nazionale di Alta Matematica Francesco Severi* (Indam-GNCS)

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# FRONTIER DATA ANALYSIS TECHNIQUES AND THEIR APPLICATIONS

TATYANA ARSHINOVA

*Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

E-mail: tatjana.arsinova@rtu.lv

The remaining uncertainty in development of Latvian national economy and the absence of significant improvement of economical situation impacts activities of all economical subjects. Due to this fact, credit institutions that are performing redistribution of income inside of the country are especially vulnerable. In June 2010 21 banks and seven branches of foreign banks were functioning in Latvia that is indicative of increasing level of competition in the banking sector. Due to the absence of improvement in quality of credit portfolio, the volumes of reserves for non-performing loans in the first current half-year have reached 1,671 million LVL. Operating profits of Latvian banks continue decreasing, mainly because of credit impairments that made 49,3% of total banking losses (430,3 million LVL) in the end of June 2010. These negative macroeconomical trends that have impact on activities of credit institutions are indicative of necessity of strong control over banking performance. Actually the estimation of operating efficiency level in the most Latvian banks is realized on the basis of quantitative approach of ratio analysis. Ratios measure the relationship between two variables chosen to provide insights into different aspects of the banks multifaceted operations, such as liquidity, profitability, capital adequacy, asset quality, risk management, and many others. Although the traditional ratio measures are attractive to analysts due to their simplicity, there are several limitations that must be considered. For example, the analysis assumes comparable units, which implies constant returns to scale (Smith 1990). Each of the indicators yields a one-dimensional measure by examining only a part of the organization's activities, or combining the multiple dimensions into a single, unsatisfactory number. Moreover, the seemingly unlimited number of ratios that can be created from financial statement data are often contradictory, thus ineffective for the assessment of overall performance. This overly simplistic analytical approach offers no objective means of identifying inefficient units and requires a biased separation of the inefficient and efficient levels. Methods of frontier analysis ensure a principally different approach to the problem of efficiency measurement. They provide an opportunity of complex analysis of banking efficiency level for a certain period of time and comparison of it among investigated banks. This multidimensional approach meets the requirements to the banking performance evaluation methodology. The objective of the author‘s research is to improve and supplement the methodology of efficiency measurement of Latvian banks on the basis of methods of frontier analysis. In the circumstances of unstable macroeconomical environment and competition, profitability is one of the most important indicators of stability and development of credit institutions. In this connection, the author analyzed the performance of a set of Latvian banks, assuming operating profits as an output. The objects of the research are members of Latvian banking sector; their efficiency level is analyzed over the time period from 2003 to 2009. Evaluating the performance on the basis of frontier approaches, the author included into the set of investigated objects banks that take leading positions on Latvian market (according to the volumes of total assets): JSC “Swedbank“, JSC “DnB Nord Banka“, JSC “Aizkraukles Banka“, JSC “Parex banka“ (currently JSC “Citadeles Banka“), JSC “SEB Banka“, JSC “Latvijas Krājbanka“, JSC “Mortgage Bank“, JSC “Rietumu Banka“, JSC “Norvik Banka“, JSC “GE Money Bank“ with the exception of branches of foreign banks.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# COMPUTATIONAL MODELING OF BIOSENSORS UTILIZING PARALLEL SUBSTRATES CONVERSION[1]

VYTAUTAS AŠERIS[1], ROMAS BARONAS[1] and JUOZAS KULYS[2]

[1] *Faculty of Mathematics and Informatics, Vilnius University*

Naugarduko 24, Vilnius LT-03225, Lithuania

E-mail: Vytautas.Aseris@mif.vu.lt, Romas.Baronas@mif.vu.lt

[2] *Department of Chemistry and Bioengineering, Vilnius Gediminas Technical University*

Saulėtekio av. 11, Vilnius LT-10223, Lithuania

E-mail: Juozas.Kulys@fm.vgtu.lt

Biosensors are analytical devices used for the recognition of the chemical substances based on biological material. Biosensors containing biological catalysts (enzymes) are called catalytical biosensors. They are widely applied in medicine, ecology and environmental monitoring [1].Some advanced biosensors involve complex bioelectrocatalytical schemes including cyclic, consecutive multi-step and parallel conversion of substrates [2]. The understanding of the kinetic peculiarities of the biosensors action is of crucial importance for their design. The mathematical modeling is widely used to improve the efficiency of the biosensors design and to optimize their configuration [3]. The purpose of this work was to develop mathematical as well as the corresponding numerical models of the biosensors utilizing parallel substrates conversion and to investigate the peculiarities of the biosensors response. We consider the following kinetic scheme of two substrates ($S_1$ and $S_2$), two enzymes ($E_1$ and $E_2$) and two reaction products ($P_1$ and $P_2$):

$$S_1 \xrightarrow{E_1} P_1, \quad S_1 + S_2 \xrightarrow{E_2} P_2.$$

The developed mathematical model is based on a system of non-linear reaction-diffusion equations. The model involves three regions: the enzyme layer, a diffusion limiting region and a convective region. In order to define the main governing parameters of the model, the corresponding dimensionless model was derived.The digital simulation was carried out by applying Crank-Nicolson finite difference method [3]. The influence of the model parameters on the biosensors behavior was numerically investigated.

**REFERENCES**

[1] A. Sadana and N. Sadana. *Handbook of Biosensors and Biosensor Kinetics*. Elsevier, Amsterdam, 2010.

[2] J. Kulys, V.V. Sorochinski and R. Vidziunaite. Transient response of bienzyme electrodes. *Biosensors*, **2** (3):135–146, 1986.

[3] R. Baronas, F. Ivanauskas and J. Kulys. *Mathematical modeling of biosensors*. Springer, Dordrecht, 2010.

[1] This research was funded by a grant (No. VP1-3.1-ŠMM-07-K-01-073) from the Research Council of Lithuania.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON SMOOTHING PROBLEMS UNDER ADDITIONAL RESTRICTIONS[1]

SVETLANA ASMUSS[1,3], NATALJA BUDKINA[2,3] and JURIS BREIDAKS[1]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

[3] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: svetlana.asmuss@lu.lv, budkinanat@gmail.com, juris.breidaks@csb.gov.lv

Let $X$, $Y$ be Hilbert spaces and assume that linear operator $T\colon X \to Y$, linear functionals $l_j\colon X \to \mathbb{R}$, $j = 1, \dots, m$, and $k_i\colon X \to \mathbb{R}$, $i = 1, \dots, n$, are continuous, functionals $l_1, \dots, l_{m-q}, k_1, \dots, k_n$ are linear independent and

$$l_{m-q+j} = \sum_{i=1}^{n} \phi_{ji} k_i,\ j = 1, \dots, q.$$

For given vectors $u = (u_1, \dots, u_{m-q})$ and $v = (v_1, \dots, v_n)$, parameters $\varepsilon_i > 0, \omega_i > 0, i = 1, \dots, n$, and matrices $\Omega = diag(\omega_i)_{i=1,\dots,n}$, $\Phi = (\phi_{ji})_{j=1,\dots,q;i=1,\dots,n}$ we consider two following conditional minimization problems:

Problem 1. (*the smoothing problem with obstacles*)

$$\|Tx\| \longrightarrow \min_{\substack{x \in X,\\ A_1 x = (u, \Phi v),\\ |(A_2 x)_i - v_i| \le \varepsilon_i,\ \ i = 1, \dots, n,}}$$

Problem 2. (*the smoothing problem with weights*)

$$\|Tx\|^2 + \|\Omega^{-1}(A_2 x - v)\|^2 \longrightarrow \min_{\substack{x \in X,\\ A_1 x = (u, \Phi v).}}$$

Here the restrictions given by $A_1 = (l_1, \dots, l_m)$ describe the interpolating conditions and the restrictions given by $A_2 = (k_1, \dots, k_n)$ describe the smoothing conditions. This talk is devoted to the analysis of Problem 1 and Problem 2 in the case when some of functionals $l_j$, $j = 1, \dots, m$, depend on $k_i$, $i = 1, \dots, n$.

[1] This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and 2009/0138/1DP/1.1.2.1.2/09/IPIA/VIAA/004 of the European Social Fund and by the grant 09.1570 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON PERIODIC SOLUTIONS OF LIÉNARD TYPE EQUATIONS[1]

SVETLANA ATSLEGA[1] and FELIX SADYRBAEV[2]

[1] *Latvia University of Agriculture*

Lielā iela 2, Jelgava, LV-3001, Latvia

E-mail: svetlana.atslega@llu.lv

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: felix@latnet.lv

The existence of isolated and non-isolated periodic solutions indifferential equations of the type

$$x'' + f(x, x')x' + g(x) = 0$$

is discussed. The existence and bifurcation results are presented, as well as the related examples are constructed.

**REFERENCES**

[1] M. Sabatini. Liénard limit cycles enclosingperiod annuli, or enclosed by period annuli. *Rocky Mount. J. Math.*, **35** (1): 253–266, 2005.

[2] M. Sabatini. On the period function of$x'' + f(x)x'^2 + g(x) = 0$. *J. Diff. Equations*, **196** : 151–168, 2004.

[3] S. Atslega, F. Sadyrbaev. Period annuli in theLiénard type equation. *International Journal of Pure and Applied mathematics*, **44** (1):117–123, 2008.

[4] L. Perko. *Differential Equations and DynamicalSystems*. 3rd ed., *Texts in Applied Mathematics vol. 7* Springer, 2001.

[1] This work was partially supported by the Latvian Council of Science research project 09.1220

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SOME PROBLEMS OF SECOND-ORDER RATIONAL DIFFERENCE EQUATIONS[1]

MARUTA AVOTIŅA[1] and INESE BULA[1,2]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: marutaavotina@inbox.lv, ibula@lanet.lv

We consider the second-order rational difference equation

$$x_{n+1} = \frac{\alpha + \beta x_n + \gamma x_{n-1}}{A + B x_n + C x_{n-1}},\ n = 0, 1, 2, \dots , \tag{1}$$

where the parameters $\alpha$, $\beta$, $\gamma$, $A$, $B$, $C$ and the initial conditions $x_{-1}$ and $x_0$ are nonnegative real numbers. These equations have been researched in the book [1]. The authors investigate the global stability character, the periodic nature, and the boundedness of solutions of a rational difference equation (1). The techniques and results that have been developed by the authors of the book [1] are developed to understand the dynamics of equation (1) are extremely useful in analyzing the equations in the mathematical models of various biological systems and other applications. It is an amazing fact that equation (1) contains, as special cases, a large number of equations whose dynamics have not been thoroughly explored yet.

In our talk we discuss about Open Problem 2.9.4 from the book [1]:

**Open Problem.** It is known that every positive solution of each of the following the equations

$$\begin{aligned} x_{n+1} &= 1 + \tfrac{x_{n-1}}{x_n}, \quad n = 0, 1, 2, \dots , \\ x_{n+1} &= \tfrac{1+x_{n-1}}{1+x_n}, \quad n = 0, 1, 2, \dots , \\ x_{n+1} &= \tfrac{x_n + 2x_{n-1}}{1+x_n}, \quad n = 0, 1, 2, \dots , \end{aligned} \tag{2}$$

converges to a solution with period-two: $\dots, \phi, \psi, \phi, \psi, \dots$. In each case, determine $\phi$ and $\psi$ in terms of the initial conditions $x_{-1}$ and $x_0$. Conversely, if $\dots, \phi, \psi, \phi, \psi, \dots$ is a period-two solution for one of equations (2), determine all initial conditions $(x_{-1}, x_0) \in ]0, +\infty[\times]0, +\infty[$ for which the solution $(x_n)_{n=-1}^{\infty}$ converges to the period-two solution.

## REFERENCES

[1] M.R.S. Kulenovič, G. Ladas. *Dynamics of Second Order Rational Difference Equations. With Open Problems and Conjectures*. Chapman&Hall/CRC, USA, 2002.

[1] This work was partially supported by ESF research project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and by the Latvian Council of Science research project 09.1220

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# COMPUTATIONAL MODELLING OF THE BACTERIAL SELF-ORGANIZATION IN A CIRCULAR CONTAINER[1]

R. BARONAS[1], Ž. LEDAS[1] and R. ŠIMKUS[2]

[1] *Faculty of Mathematics and Informatics, Vilnius University*

Naugarduko 24, LT-03225 Vilnius, Lithuania

[2] *Vilnius University Institute of Biochemistry*

Mokslininku 12, LT-08662 Vilnius, Lithuania

E-mail: romas.baronas@mif.vu.lt, remigijus.simkus@bchi.vu.lt, zilvinas.ledas@dict.lt

The directed movement of microorganisms in response to chemical gradients is called chemotaxis. Chemotaxis plays a crucial role in a wide range of biological phenomena. Particularly, it is important for bacteria to find food by swimming towards the highest concentration of food molecules [1].

This paper focuses on computational modelling of a bacterial self-organization in a circular container as detected by bioluminescence imaging [2]. Over the last decade, lux-gene engineered bacteria have been used to develop whole cell-based biosensors [3]. The pattern formation in a luminous *Escherichia coli* colony was modelled by the nonlinear reaction-diffusion-chemotaxis equations,

$$\begin{aligned} \frac{\partial n}{\partial t} &= D_n \Delta n - \nabla \left( \frac{k_1 n}{(k_2 + c)^2} \nabla c \right) + k_3 n \left( 1 - \frac{n}{n_0} \right), \\ \frac{\partial c}{\partial t} &= D_c \Delta c + \frac{k_4 n^p}{k_5 + n^p} - k_6 c, \quad \mathrm{x} \in \Omega, \quad t > 0, \end{aligned} \tag{1}$$

where $\Omega$ stands for the circular domain, $n(\mathrm{x}, t)$ is the cell density, $c(\mathrm{x}, t)$ is the chemoattractant concentration, $p$ stands for the dependence order of the attractant production, $p = 1, 2$ [4]. The governing equations (1) together with appropriate initial, boundary and matching conditions form a boundary-value problem, which was numerically solved by applying the finite difference technique.

By varying the input parameters the output results were numerically analyzed with a special emphasis to the influence of the model parameters on the pattern formation. The numerical simulation at transition conditions was carried out using the finite difference technique. The mathematical model and the numerical solution were validated by experimental data [2].

**REFERENCES**

[1] T.C. Williams. *Chemotaxis: Types, Clinical Significance, and Mathematical Models*. Nova Science, New York, 2011.

[2] R. Šimkus, V. Kirejev, R. Meškienė and R. Meškys. Torus generated by *Escherichia coli*. *Experiments in Fluids*, **46** (2):365–69, 2009.

[3] R. Daniel, R.Almog and Y. Shacham-Diamand. Characterization methods of a whole-cell bioluminescent biosensor. *IEEE Sensors Journal*, **10** (2):274–280, 2010.

[4] T. Hillen and K.J. Painter. A users guide to PDE models for chemotaxis. *Journal of Mathematical Biology*, **58** (1-2):183–217, 2009.

[1] This research was funded by the Global Grant (No. VP1-3.1-ŠMM-07-K-01-073) from the Research Council of Lithuania. This research was performed in cooperation with Vilnius University.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON THE REGULARITIES OF CRITICAL REGIMES OF TWO-PHASE VERTICAL FLOWS

MICHAEL BARSKY[1] and EUGENE BARSKY[2]

[1] *Ben Gurion University of the Negev, Beer Sheva, Israel*
[2] *Jerusalem College of Engineering, Israel*

The present paper is an attempt to apply partial differential equations similar to Navier-Stokes equations to the process of classification. Using these equations, we have substantiated the Froude criterion for modeling mass transfer in turbulent regimes of critical flows. Similarity criteria for other regimes (laminar and transient ones) are also obtained by solving this equation. Using these criteria, we have substantiated universal curves characterizing fractional extraction of particles of various size classes in such flows. The general character of these criteria and their adequacy to experimental data are demonstrated.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# EXPLICIT NORDSIECK METHODS

ZBIGNIEW BARTOSZEWSKI[1] and ZDZISŁAW JACKIEWICZ[2,3]

[1] *Faculty of Applied mathematics and Physics, Gdańsk University of Technology*

G. Narutowicza 11/12, 80-289, Gdańsk, Poland

[2] *Department of Mathematics, Arizona State University*

Tempe, Arizona, 85287, USA

[3] *AGH University of Science and Technology,Kraków, Poland*

al. Mickiewicza 30, 30-059 Kraków, Poland

E-mail: zbart@pg.gda.pl, jackiewi@math.la.asu.edu

In our talk we will describe the construction of explicit Nordsieck methods of order $p$ and stage order $q = p$ for $p = 1, 2, 3, 4$ with large regions of absolute stability. Error propagation and estimation of local discretization errors will also be presented. The error estimators for examples of general linear methods constructed recently by the authors will be included in our talk and the results of some numerical experiments will illustrate the effectiveness of proposed methods.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# DYNAMICS OF FLEXIBLE MAGNETIC MICRORODS[1]

MIHAILS BELOVS[1], ANDREJS CEBERS[1] and HARIJS KALIS[1,2]

[1] *Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, LV-1002, Rīga, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: mihails.belovs@lu.lv, andrejs.cebers@lu.lv, kalis@lanet.lv

The mechanisms of self-propulsion of microorganisms and dynamics of flexible filaments have obtained considerable interest recently [1].

Rather promising approach for the creation of different microdevices which mimics the mechanisms used by living organisms is application of flexible magnetic microrods [2; 3].

Here we review the present state of the art of this field. Principles of the construction of models of magnetic filaments are described and the numerical algorithms for the solution of corresponding nonlinear partial differential equations are considered.

The general principles and algorithms are illustrated by numerical solutions of the self-propelling motion of superparamagnetic and ferromagnetic filaments, the loop formation by the ferromagnetic filament at field inversion, anomalous orientation of a ferromagnetic filament under the action of an AC field and behavior of magnetic filaments under the action of a rotating field.

The comparison of the results of numerical solution with physical eksperiments is given.

**REFERENCES**

[1] T.R.Powers. Dynamics of filaments and membranes in a viscous fluid. *Reviews of Modern Physics*, **82** :1607, 2010.

[2] R.Dreyfus, J.Baudry, M.L.Roper, M.Fermigier, H.A.Stone and J.Bibette. Microscopic artificial swimmers. *Nature*, **437** :862, 2005.

[3] A.Cebers. Flexible magnetic filaments. *Current Opinion in Colloid and Interface Science*, **10** :167, 2005.

[1]This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund and by the grant 09.1572 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ARMA FITTING FOR LATVIAN GDP

ALEKSANDRS BEZRUCKO

*Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

E-mail: aleksandrs.bezrucko@rtu.lv

The analysis and forecast of GDP is important task for economists, policy makers and entrepreneurs. These researches are consisting of many objective and subjective factors. In econometrics forecast, not only statistical methods are used, but a lot of economical and political events must be taken into account. In this work the algorithm is developed for finding optimal time series model for GDP forecasting. Latvian GDP data with quarterly observation frequency is taken as time series. ARMA Analysis of Latvian GDP time series is performed. The set of model has been constructed. In order to check the accuracy of models, different residual tests are performed: autocorrelation, heteroscedasticity and normality of residual distribution. Models are compared in their forecast quality. Working on the paper, different methods of econometrical modelling have been analyzed. For example, analysis methods for German GDP forecast that are described by Lutkepohl in "Applied Time Series Analysis". Lutkepohl described different ways of ARMA and Residual analysis of time series. In this paper author uses familiar methods of statistical analysis of time series for forecasting Latvian GDP. Computer software enabled the author to perform the search for the best models for certain time series. Based on the analysis of these models, a search algorithm of optimal model is created. In order to find an optimal model of forecasting Latvian Gross Domestic Product, two different cases of Latvian GDP series with quarterly observation frequency are taken. The first case is Latvian quarterly GDP series in levels (Latvian lats) and second case is the same data in percentage growth. The GDP series are given in Figure 1. The time series length is $T = 60$. The time series is taken from the first quarter of year 1996 till the fourth quarter of year 2009. All searches and forecasts are made using econometrical software EViews 6.0.

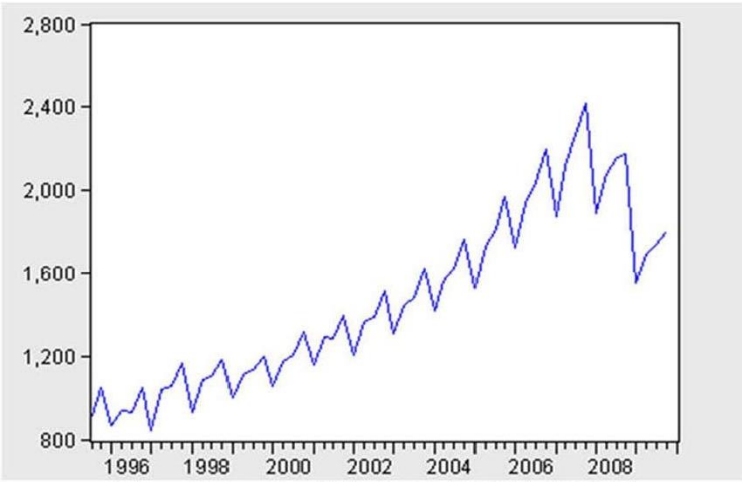


**Figure 1.** Latvian GDP (Lats) 1995Q1 – 2009Q4

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# STRICTLY CONVERGENT ALGORITHM FOR AN ELLIPTIC EQUATION WITH NONLOCAL AND NONLINEAR BOUNDARY CONDITIONS[1]

K. BIRGELIS and U. RAITUMS

*Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: uldis.raitums@lumii.lv

We consider an elliptic equation with nonlinear and nonlocal boundary conditions, which arises in conductive-radiative heat transfer problems, see, for instance, [1; 2; 3]. The corresponding to our problem variational equality reads as

$$\begin{array}{l}\int_\Omega \big[k_1\langle\nabla(u+u_*),\nabla\eta\rangle + k_2(u+u_*)_{x_3}\eta\big]\,dx \\ \quad + \int_\Gamma \sigma\big[(I-H)(\mid u+u_*\mid^3 (u+u_*))\big]\eta\,dS \\ \quad = \int_\Omega \langle\overline{f},\eta\rangle\,dx + \int_\Gamma g\eta\,dS \;\;\forall\eta\in V,\end{array} \tag{1}$$

where $\Omega = \Sigma\times[0,L]\subset\mathbf{R}^3$ is a bounded cylinder , $V$ is a subspace of $W_2^1(\Omega)$ of functions that are zero on the intersection of $\overline{\Omega}$ with the plane $\{x_3=0\}$, $\Gamma$ is the latarel surface of $\Omega$, $k_1,k_2,\sigma$ are positive constants, but $H$ is a nonlocal bounded linear operator from $L_p(\Gamma)$ to $L_p(\Gamma)$ such that for $p=1$ its norm is less than 1.

We show that there exists a two level iterative process that converges to the solution of (1). The first level consists of the Newton-type process

$$\begin{array}{l}\int_\Omega \big[k_1\langle\nabla v_{k+1},\nabla\eta\rangle + v_{k+1x_3}\eta\big]\,dx + \int_\Gamma \sigma\psi(v_k)v_{k+1}\,dS \\ = \langle\langle F(v_k),\eta\rangle\rangle \;\;\forall\eta\in V,\;\; k=1,2,\dots\,,\end{array}$$

with approriate nonnegative function $\psi$ and $F(v_k)\in\big(V\big)^*$. In its turn, the second level consists on iterations of the type

$$\begin{array}{l}\int_\Omega \big[k_1\langle\nabla(u_{k+1}+u_*),\nabla\eta\rangle + k_2(u_{k+1}+u_*)_{x_3}\eta\big]\,dx + \int_\Gamma \sigma\big[\mid u_{k+1}+u_*\mid^3 (u_{k+1}+u_*)\big]\eta\,dS \\ = \int_\Gamma \sigma H\big[\mid u_k+u_*\mid^3 (u_k+u_*)\big]\eta\,dS + \langle\langle F_0,\eta\rangle\rangle \;\;\forall\eta\in V,\;\; k=1,2,\dots\,,\end{array}$$

with an appropriate $F_0\in\big(V\big)^*$.

## REFERENCES

[1] K.Birgelis. Optimal control in models with conductive-radiative heat transfer. *Mathematical Modelling and Analysis*, **8** 1:1-12, 2003.

[2] P.E.Durot, O.Klein, J.Sprekels, F.Troeltzsch and I.Yousept. Optimal control of 3D-state-constrained induction heating problem with nonlocal radiaton effect. In: *Weierstraß institute for Angevandte Analysis und Stochastic*, Preprint No. 1422 , 2009.

[3] M.Laitinen and T.Tiihonen. Heat transfer by conduction and radiation in grey materials. In: *University of Jyvaskyla*, Report No. B6/2000, 2000.

[1]This work was supported by the Latvian Council of Sciences under grant 09.1572 and by the ESF project No. 2009/0223/IDP/1.1.1.2.0/APIA/VIAA/008.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# STABILIZED FINITE DIFFERENCE METHODS FOR THE FULLY DINAMIC BIOT'S PROBLEM

N. BOAL, F.J. GASPAR, F.J. LISBONA and P.N. VABISHCHEVICH

*Zaragoza University*
Department of Applied Mathematics
María de Luna 3
50018 Zaragoza
Spain

E-mail: nboal@unizar.es

Great interest in geomechanics, hydrogeology and biomechanics is focused on the time-dependent coupling between the deformation of a porous matrix and the fluid flow inside. Biot studied this interaction for a homogeneous porous medium and, in 1956, he proposed the mathematical equations governing the dynamic behavior of fully saturated porous media, as a simple extension of his earlier work on quasi-static case.

This model can be given by a system of partial differential equations for the displacements of the solid phase $\mathbf{u}(\mathbf{x},t)$ and the pressure of the fluid $p(\mathbf{x},t)$ as the essential variables. So, let $\Omega$ be a domain in $\mathbb{R}^n$, $n \leq 3$, occupied by an elastic, porous and permeable matrix of density $\rho$, saturated by a viscous and slightly compressible fluid. The corresponding equations read as

$$\begin{aligned} \rho\frac{\partial^2\mathbf{u}}{\partial t^2} - \mu\Delta\mathbf{u} - (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) + \alpha\,\nabla p &= \mathbf{g}(\mathbf{x},t), \\ \frac{\partial}{\partial t}\left(\gamma\, p + \alpha\nabla\cdot\mathbf{u}\right) - \nabla\cdot\left(\frac{\kappa}{\eta}\,\nabla p\right) &= f(\mathbf{x},t), \quad \mathbf{x}\in\Omega,\ 0<t<T, \end{aligned}$$

where $\lambda$ and $\mu$ are the Lamé coefficients; $\gamma = n\tilde{\gamma}$, with $n$ the porosity and $\tilde{\gamma}$ the compressibility coefficient of the fluid; $\kappa$ is the permeability of the porous medium, $\eta$ is the viscosity of the fluid and $\alpha$ is the Biot-Willis constant.

A very interesting case of this mixed hyperbolic-parabolic system corresponds to the case of an incompressible fluid with small permeability, due to the presence of nonphysical oscillations in the numerical solution if standard discretizations are considered.

To overcome this difficulty, by one hand we propose a family of finite-difference schemes on staggered grids for space. A priori estimates in discrete composed norms are obtained, convergence results are given and numerical experiments confirming the theoretical results are presented. And by other hand, we also propose stabilized finite difference schemes on collocated grids based on the perturbation of the equation for the displacements, which permit us obtain solutions free of oscillations independently of the chosen discretization parameters.

This research has been funded by DGA and project MTM 2010-16917.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# A MATHEMATICAL MODEL FOR A NORWEGIAN FLUTE[1]

TABITA BOBINSKA[1] and ANDRIS BUIKIS[1,2]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: tabita.bobinska@gmail.com, buikis@latnet.lv

The Norwegian folk flute, also known as willow flute (Norwegian: *seljefløyte*), is a musical instrument of the recorder family, although it is side-blown. It consists of a tube with a transverse fipple mouthpiece that provides a passageway for airflow to be directed against a sharp edge creating a tone. The flute has no finger holes, so different pitches are produced by overblowing and by using a finger to cover, half-cover or uncover the hole at the far end of the tube.

As it is well known, string instruments and many wind instruments are modelled by the wave equation (see, e.g., [1] - [5], etc.). But unlike the authors who use the homogeneous one-dimensional equation to describe sound vibrations inside the willow flute (see [1], [4]), we describe the process more accurately by adding a source term to the equation:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + f(x,t),\ 0 < x < l.$$

Here $u(x,t)$ denotes the pressure in the flute, $x$ is the distance along the tube, $t$ is time.

To state the initial-boundary value problem, the following boundary conditions are introduced:

$$u_x(0,t) - \gamma u(0,t) = 0,$$
$$(1-h)u_x(l,t) + hu(l,t) = 0,\ 0 \le h \le 1.$$

Solving the problem, we find the possible frequencies of the harmonics of sounds produced by the flute, determine the distribution of energy between the fundamental and its overtones, and analyse how the tones may be altered by changing the parameter $h$ in the boundary condition at $x = l$.

**REFERENCES**

[1] R. W. Hall and K. Josić. The Mathematics of Musical Instruments. *American Mathematical Monthly*, **108** (4): 347–357, 2001.

[2] H. Lamb. *The Dynamical Theory of Sound*. Edward Arnold, London, 1910.

[3] T. D. Rossing (Ed.). *Springer Handbook of Acoustics*. Springer Science+Business Media, LLC New York, 2007.

[4] L. Simanoviča. *Boundary Conditions Effect on Musical Instruments Timbre: Bachelor's Thesis*. Faculty of Physics and Mathematics, University of Latvia, Riga, 2010.

[5] J. W. Strutt, Baron Rayleigh. *The Theory of Sound, Vol. II*. Macmillan and Co., London, 1878.

[1] This work is partially supported by the project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and the project "Support for Doctoral Studies at University of Latvia" of the European Social Fund and by the grant 09.1572 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON SOME QUADRATURE RULES WITH LAPLACE END CORRECTIONS

BOGUSŁAW BOŻEK, WIESŁAW SOLAK and ZBIGNIEW SZYDEŁKO

*AGH University of Science and Technology, Faculty of Applied Mathematics*

Poland, 30-059 Kraków, al. Mickiewicza 30

E-mail: bozek@uci.agh.edu.pl

How can one compute the sum of an infinite series $s := a_1 + a_2 + \cdots$? If the series converges fast, i.e., if the term $a_n$ tends to 0 fast, then we can use the known bounds on this convergence to estimate the desired sum by a finite sum $a_1 + a_2 + \cdots + a_n$. However, the series often converges slowly. This is the case, e.g., for the series $a_n = n^{-t}$ $\big(t \in (1, +\infty)\big)$ that defines the Riemann zeta-function. In such cases, to compute $s$ with a reasonable accuracy, we need unrealistically large values $n$, and thus, a large amount of computation.

In many cases, the $n$-th term of the series can be obtained by applying a smooth function $f(x)$ to the value $n$: $a_n = f(n + 1/2)$. In such situations, we can get more accurate estimates if instead of using the upper bounds on the remainder infinite sum $R = f(n + 3/2) + f(n + 5/2) + \ldots$, we approximate the remainder by the corresponding integral $I$ of $f(x)$ (from $x = n+1$ to infinity), and find good bounds on the difference $I - R$.

First, we derive sixth order quadrature formulas (some quadrature rules with Laplace end corrections with a parameter $\beta$) for functions whose 6th derivative is either always positive or always negative and then we use these quadrature formulas to get good bounds on $I - R$, and thus good approximations for the sum $s$ of the infinite series. The main result of the talk can be expressed in the

THEOREM 1. *We assume that the function $f$ is such that: $f$ is either positive and decreasing, or negative and increasing on $[1, +\infty)$, $\int_1^\infty f(x)dx$ is convergent, $f \in C^6\big([1, \infty)\big)$, $f^{(6)}$ is either positive or negative on $\big[1, \infty\big)$. We define $a_j := f(j + 1/2)$, $j \in \mathbb{N}$, $s := \sum_{n=0}^{\infty} a_n$. Under this assumptions, if $f^{(6)} > 0$ then:*
$\sum_{j=0}^{m-1} a_j + \int_m^\infty f(x)\,dx - P_m(\sqrt{\frac{7}{230}}, f) < s < \sum_{j=0}^{m-1} a_j + \int_m^\infty f(x)\,dx - P_m(-\sqrt{\frac{7}{230}}, f)$
*for $m > 2$, where* $P_m(t, f) := \frac{115}{84} t \left(2f(m + \frac{1}{2}t) - 3f(m + \frac{3}{2}t)\right) + f(m + \frac{5}{2}t)\Big)$.

It is continuation of the results from [3], [2], [1].

**REFERENCES**

[1] B. Bożek B., W. Solak, Z. Szydełko. On some quadrature rules with Gregory end corrections. *Opuscula Mathematica*, **29** 2, 117–129 (2009).

[2] W. Solak. A remark on power series estimation via boundary corections with parameter. *Opuscula Mathematica*, **19** 75–80, (1999).

[3] W. Solak, Z. Szydełko. Quadrature rules with Gregory–Laplace end corrections. *Journal of Computational and Applied Mathematics*, **36** 251 – 253, (1991).

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# EXPLICIT NORDSIECK METHODS WITH QUADRATIC STABILITY

MICHAŁ BRAŚ

*Faculty of Applied Mathematics, AGH University of Science and Technology*

al. Mickiewicza 30, 30-059 Kraków, Poland

E-mail: bras@agh.edu.pl

We describe the search for explicit Nordsieck methods with large region of absolute stability. These methods are given by abscissa vector $\mathbf{c} \in \mathbb{R}^s$ and four coefficient matrices $\mathbf{A} \in \mathbb{R}^{s\times s}$, $\mathbf{U} \in \mathbb{R}^{s\times r}$, $\mathbf{B} \in \mathbb{R}^{r\times s}$, and $\mathbf{V} \in \mathbb{R}^{r\times r}$. We are interested in methods of order $p$ and stage order $q$ for which $p = q = s$ and $r = s+1$. Using theory of order for general linear methods [3] we can express matrices $\mathbf{U}$ and $\mathbf{B}$ in terms of $\mathbf{A}$, $\mathbf{V}$, and $\mathbf{c}$. Then remaining parameters are used to ensure quadratic stability (i.e. that the stability function of this method has only two non-zero roots [1; 2]) and maximize region of absolute stability. Obtained methods might be useful for solving nonstiff systems of ordinary differential equations in variable order and variable stepsize pattern.

**REFERENCES**

[1] M. Braś. Nordsieck Methods with Inherent Quadratic Stability. *to appear in Math. Model. Anal*, **16** .

[2] D. Conte and R. D'Ambrosio and Z. Jackiewicz. Two-step Runge-Kutta methods with quadratic stability functions. *J. Sci. Comput.*, **44** (2):191–218, 2010.

[3] Z. Jackiewicz. *General Linear Methods for Ordinary Differential Equations*. John Wiley & Sons, Hoboken, New Jersey, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ISSUES OF HYPERBOLIC HEAT EXCHANGE EQUATION IN POLAR CO-ORDINATE SYSTEM[1]

ANDRIS BUIĶIS[1,2] and ANITA PILIKSERE[2]

[1] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

[2] *Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: buikis@latnet.lv, anita.piliksere@gmail.com

Intensive quenching method in water is patented several decades ago [1; 2]. Classical heat exchange equation [3; 4] is used to describe the process. To describe quenching process by hyperbolic heat exchange equation in paper [5] by prof. A.Buiķis is offered.

Intensive quenching technical processes as well as mathematical models meet with several difficulties caused by a number of reasons. Firstly, if cooling process of part exceeds some critical heat flow value thin layer of water steam can form on the surface of part and so called film boiling can begin. During film boiling process part can be frequently fractured. Then nucleate boiling process caused by fractures begins, id est., non linear boundary conditions required for process mathematical describing. Secondly, there are no exact mathematical verified solutions for parts with more complicated geometry. Usually Kondratjev number (form factor) used to reduce complex problem to problem with simpler domain. Thirdly, solution of hyperbolical heat exchange equation requires additional initial conditions (heat flux), which is unknown from praxis; therefore issue is reduced to inverse non stationary problem.

All these issues are discussed in this talk.

**REFERENCES**

[1] N.I.Kobasko. Quenching of Steel in Liquid Media Under Pressure. *Naukova Dumka, Kiev*, 1980, 206 p..

[2] N.I.Kobasko. *Intensive Steel Quenching Methods, Handbook Theory and Technology of Quenching*. Springer-Verlag, 1992.

[3] N.I.Kobasko, V.Dobrivecher. Self-Regulated Thermal Process and Cooling Properties of Quenchants. In: *IASME Transactions*, Vol. 2, November 2005, Issue 9, pp. 1825-1828.

[4] N.I.Kobasko, A. Buiķis. Cooling time calculation for any configuration of steel part. In: *Proceedings of 9th International Scientific and Technical Congress of Heat-Treaters and Metallographers*, Krakov, Vol. I, 2008, pp. 275-281.

[5] A.Buiķis, Sh. Guseinov. Solution of reverse hyperbolic heat equation for intensive carburized steel quenching. In: *Proceedings of 13th International Conference on Computational and Experimental Engineering and Sciences*, Madras, India, December 1-6, 2005, pp. 741-746.

[1] This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and 2009/0138/1DP/1.1.2.1.2/09/IPIA/VIAA/004 of the European Social Fund and by the grant 09.1572 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON MARKET PRICE CYCLE BIFURCATION

MĀRIS BUIKIS

*Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

E-mail: maris.buikis@rtu.lv

The paper deals with stochastic analysis of price dynamics at a single-component market. Our model supposes that a manufacturer has a monopoly there and he would like to stabilize the price of a product unit into a small neighborhood of the level $\bar{p}$. Let us remind, that in any classical single market model a price equilibrium $p(t) \equiv \bar{p}$ can be achieved by the equality of demand $D(\bar{p})$ to supply $S(\bar{p})$ (see, for example [1] and [3]). To control a price at the time moment $t$ the manufacturer can use a supplied quantity $S_t$, but to enter the market he needs some time $h$ (assumption of Marshall [1]).Therefore manufacturer has a delayed reaction because guides by the price at the moment of time $t-h$. Our approach is based on stochastic asymptotic methods of bifurcation theory for random dynamical systems with delay [4]. After analysis of linear model we will pass to more complicated stochastic nonlinear dynamics model, writing an adaptive market assertion [5] in a form of stochastic Ito equation

$$dp(t) = (S_t - D_t)dt + \sigma p(t)dw(t), \tag{1}$$

with nonlinear demand $D_t = D(p(t))$, delayed supply $S_t = ap(t-\tau(t)) + b$, where $\tau(t)$ is stationary Markov process, $w(t)$ is Wiener process, and parameter $\sigma$ (called by *volatility*) allows to take into account value of risk. This permits to detect stable stationary price cycle on the Demand-Supply-plane with frequency $\nu$ defined by equation $\nu = b\Im\chi(\nu)$, where $\chi(\nu)$ is characteristic function of invariant distribution of $\tau(t)$, and random amplitude, satisfying stochastic differential equation Ornstein-Ulenbeck type. Proposed by our paper mathematical model of price stochastic dynamic allows not only to advance in price equilibrium stability analysis but also to prove that random time delay of supply exerts significant influence on market price dynamics.

**REFERENCES**

[1] A.Marshall. *Principles of Economics*. Macmillan Press Ltd., NY, 1979.

[2] R.Merton. *Contnuous-Time Finance*. Blackwell, Cambridge, UK, 1990.

[3] P.A.Samuelson. *Foundations of Economic Analysis*. Atheneum, NY, 1974.

[4] J. F. Carkov. *Random Perturbations of Functional Differential Equations*. Zinatne, Riga, 1989, (in Russian).

[5] W.Sharpe. Capital Asset Prices: a Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance*, September: 425–442,1964.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# A BAND PASS FILTER FOR REAL-TIME SIGNAL EXTRACTION

GINTERS BUSS

*Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

E-mail: ginters.buss@rtu.lv

The paper proposes an extension of the symmetric Baxter-King band pass filter to an asymmetric Baxter-King filter. In order to optimally approximate an ideal symmetric linear filter in a Baxter-King sense, the problem is to minimize

$$Q = \int_{-\pi}^{\pi} \delta(\omega)\delta(-\omega)\, d\omega \tag{1}$$

s.t.

$$\hat{\beta}(0) = \sum_{h=-p}^{f} \hat{b}_h = 0, \tag{2}$$

where $\delta(\omega) = \beta(\omega) - \hat{\beta}(\omega)$ is the discrepancy between frequency response functions of the exact and the approximate filters at frequency $\omega$, and the constraint (2) is to ensure zero weight on the trend frequency.

The FOCs are $-4\pi[b_h - \hat{b}_h] - \lambda = 0$, where $\lambda$ is Lagrangian multiplier. If there is no constraint (2), the optimal approximate (in Baxter-King sense) filter is simply derived by truncation of the ideal filter's weights. If there is a constraint (2), then rewrite FOCs as $\hat{b}_h = b_h + \theta$, where $\theta = \lambda/(4\pi)$. In order to have $\hat{\beta}(0) = \sum_{h=-p}^{f} \hat{b}_h = 0$, the required adjustment is $\theta = -\sum_{h=-p}^{f} b_h/(p+f+1)$, which yields the same optimal weight adjustment scheme as in the symmetric Baxter-King filter case, i.e, cut the ideal filter at the appropriate length and add a constant to all filter weights to ensure zero weight on zero frequency. Since the symmetric Baxter-King filter is unable to extract the desired signal at the very ends of the series, the extension to an asymmetric filter is useful whenever the real time estimation is needed.

The paper uses Monte Carlo simulation to compare the proposed filter's properties in extracting business cycle frequencies to the ones of the original Baxter-King filter and Christiano-Fitzgerald filter. Simulation results show that the asymmetric Baxter-King filter is superior to the asymmetric default specification of Christiano-Fitzgerald filter in real time signal extraction exercises.

**REFERENCES**

[1] M. Baxter and R. G. King. Measuring business cycles: approximate band-pass filters for economic time series. *The Review of Economics and Statistics*, **81** (4):575–593, 1999.

[2] L. J. Christiano and T. J. Fitzgerald. The band pass filter. *International Economic Review*, **44** (2):435–465, 2003.

[3] M. B. Priestley. *Spectral analysis and time series*. Academic Press, Inc., 1981.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# CONTINUOUS NUMERICAL METHODS FOR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

ANGELAMARIA CARDONE

*Dipartimento di Matematica, Università di Salerno*

via Ponte don Melillo, I-84084 Fisciano (Salerno), ITALY

E-mail: ancardone@unisa.it

The aim of our research is the construction of efficient and accurate numerical methods for the solution of Volterra Integro-Differential Equations (VIDEs). In order to increase the order of convergence of classical one-step collocation methods, we propose multistep collocation methods, which have been successfully introduced for Volterra integral equations in [1; 2]. Moreover, they are continuous methods, i.e. they furnish an approximation of the solution at each point of the time interval. In this talk we describe the derivation of multistep collocation methods for VIDEs and the analysis of convergence and stability properties. We show some examples of methods which compare favorably with respect to existing one-step methods. This is a joint work with B. Paternoster and D. Conte from University of Salerno.

**REFERENCES**

[1] D. Conte, Z. Jackiewicz, B. Paternoster. Two-step almost collocation methods for Volterra integral equations. *Appl. Math. Comput.*, **204** :839–853, 2008.

[2] D. Conte, B. Paternoster. Multistep collocation methods for Volterra Integral Equations. *Appl. Math. Comput.*, **59** :1721–1736, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# STOCHASTIC EXPONENT IN MATHEMATICAL FINANCE

JEVGEŅIJS CARKOVS

*Riga Technical University*

Kaļķu iela 1, Rīga, LV-1658, Latvia

E-mail: carkovs@latnet.lv

To take into account the financial market statistical uncertainty and to choose a rational strategy an investor should draw attention not only to smooth dynamical performances of financial flows, reached by moving-average procedure, but also he has to analyze extremely complicated and bad predictable chaotic financial flow oscillation. This made many researchers use Ito stochastic calculus for mathematical modeling of contemporary financial instruments. Most frequently technique in operation for mathematical finance is based on analysis of linear stochastic differential equations (see, for example, [1; 2]), among which very important is *stochastic exponent.* defined on filtered probability space $(\Omega, \mathfrak{F}, \mathfrak{F}^t, \mathbf{P})$ by formula $\mathcal{E}_Z(t) := \exp\{Z(t) - 0.5[Z]_t\}$ where $Z(t)$ is $\mathfrak{F}^t$-adopted semimartingale with quadratic variation $[Z]_t$. This process satisfies stochastic differential equation $d\mathcal{E}_Z(t) = \mathcal{E}_Z(t)dZ(t)$, that also is called a stochastic exponent. The proposal lecture is devoted to short summary of results related to *Brown stochastic exponent* (or *geometric Brown motion*) with supermartingale $Z(t) = \mu t + \sigma w(t)$ where $w(t)$ is $\mathfrak{F}^t$-adopted Wiener process. Briefly an outline of the subjects in a lecture looks in the following way. At first we will discuss how do this continuous time equations helps to modelate discrete time processes of financial market. Then we will make an introduction to option prising theory [2] using martingale-exponent (that is $\mathbf{E}\{\mathcal{E}_Z(t)/\mathfrak{F}^s\} = \mathcal{E}_Z(s)$ for $t > s$) as a Radon-Nikodym derivative for construction of market neutral probability measure $\tilde{\mathbf{P}}$ by Girsanov theorem. Now for any $\mathfrak{F}^T$-measurable payoff $F_T(S(T))$ of a derivative security based on stocks $\{S(t), 0 \le t \le T\}$ one can find pricing as conditional expectation $P(t, S(t)) = \tilde{\mathbf{E}}\{F_T(S(T))/\mathfrak{F}^t\}$ in terms of market probabilities. Owing Markov property for stochastic exponent this risk-neutral prising may be given as a solution of the backward Kolmogorov equation by classical Feynman-Kac formula. This leads to very popular and beeing of primary importance in mathematical fifnance option hedging Black-Scholes formula [1; 2]. The last part of lecture contains some of results in relation to moments of stochastic exponent, which may be useful for portfolio selection by Markovitz theory [1]. Here we derive an equation for covariances of *matrix stochastic exponent* given by matrix solution of multidimensional linear stochastic differential equation. Finally we will study a time asymptotic for *delayed stochastic exponent* defined as linear stochastic functional differential equation [3]. This financial mathematical model is used in order to take into account the past of price index [4].

**REFERENCES**

[1] David C. Shimko. *Finance in Continuous Time.A Primer.* Kolb Publishing Company, Miami, USA, 1992.

[2] Steven E. Shreve. *Stochastic Calculus for Finance II: Continuous-Time Models.* Springer Verlag, NY, 2005.

[3] J. F. Carkov. *Random Perturbations of Functional Differential Equations.* Zinatne, Riga, 1989, (in Russian).

[4] J.A.D. Appleby, X.Mao, and M.Riedle. Geometric Brownian motion with delay: Mean square characterisation. *Proceedings of the American Mathematical Society*, **137** 1: 339—348, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# INTRINSIC CURVE DYNAMICS OF MAGNETIC FILAMENTS[1]

ANDREJS CEBERS[1] and HARIJS KALIS[1,2]

[1] *Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, LV-1002, Rīga, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: `andrejs.cebers@lu.lv, kalis@lanet.lv`

Dynamics of an elongated ferromagnetic filament shape under the action of a rotating magnetic field is considered by mathematical modelling. We denote the partial derivative $\frac{\partial q}{\partial l}$ for the function $q(l,t)$ respect to space variable $l$ with $q'(t)$ ($l$ is the arclength of the center line, $t$ is the time, $L$ is the total length of filament $l \in [0,L]$). The actual shape of a droplet is obtained by solving initial-boundary value problem of following partial differential equation (PDE) [1]:

$$\frac{\partial \vec{r}}{\partial t} = -\vec{r}\,'''' - Cm\,\vec{n}\,' + (\Lambda\,\vec{t}\,)',$$

where for 2D case $\vec{r} = (x,y,0)^T$ - the configuration vector of filament, $\vec{t} = (x',y',0)^T$, $\vec{n} = -(y',x',0)^T$ - the tangent and normal vectors respectively, $Cm$ - the magnetoelastic number. The Lagrange multiplier $\Lambda$ is found using the conditions of inextensibility of filament. For the curvature of filaments $K(t,l)$ we have the system of two nonlinear PDEs

$$\frac{\partial K}{\partial t} = -K^{(4)} - 3.5K^2K'' - 3K(K')^2 - K^5 + 2\Lambda K^3 + 3\Lambda' K' + \Lambda K'',$$

$$\Lambda'' - CmK' + K(K'' + 0.5K^3) - \Lambda K^2 = 0,$$

For the discretization in space the central differences are used. Time evolution of nonstationary and coresponding of stationary solutions are obtained with MATLAB by solving large system of ordinary differential equations. The shape of filaments is obtained by solving the stationary equations of curvature. It is shown that curve dynamics of magnetic filaments may be obtaned by solving the equations for intrinsic parameters.

**REFERENCES**

[1] A.Cebers and H.Kalis. Dynamics of superparamagnetic filaments with finite magnetic relaxation time. *Eur. Phys. Journ. E*, **34** 30, 2011. DO i 10.1140/ epje/ i2011-11030-y

[1] This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund and by the grant 09.1572 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# NUMERICAL EXPERIMENTS OF SINGLE MODE GYROTRON EQUATIONS[1]

JĀNIS CEPĪTIS[1,2], OLGERTS DUMBRAJS[3], HARIJS KALIS[1,2], ANDREJS REINFELDS[1,2] and DANA CONSTANTINESCU[4]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

[3] *Institute of Solid State Physics of University of Latvia*

Ķengaraga iela 8, Rīga, LV-1063, Latvia

[4] *Department of Applied Mathematics, University of Craiova*

A.I.Cuza Street 13, Craiova 1100, (200585) Romania

E-mail: janis.cepitis@lu.lv, olgerts.dumbrajs@lu.lv, harijs.kalis@lu.lv

E-mail: reinf@latnet.lv, dconsta@yahoo.com

The present work continues our recent investigations of the stationary problem of the single mode gyrotron equation [1] using the implicit finite difference schemes and the method of lines. We consider two versions of the gyrotron equation. The amplitude $f(t,x)$ of the high frequency field in the gyrotron resonator and the transverse orbital momentum $p(T,x,\theta_0)$ of electrons can be described by the following system of two complex differential equations (new version):

$$\begin{cases} \frac{\partial p}{\partial x} + i\left(\Delta + |p|^2 - 1 - g_b\right) p = i f(t,x) \\ \frac{\partial^2 f}{\partial x^2} - i(1+\delta_\omega)\frac{\partial f}{\partial t} + (1 + 0.5(\delta_\omega + g_c)) g_d f = (1+\delta_\omega)(1+g_c)^2 I < p >, \end{cases} \tag{1}$$

where $i = \sqrt{-1}$, $x \in [0, L]$ and $t \geq 0$ are the axial and time coordinates, $L$ - the length of the interaction space, $\Delta, \delta_\omega, \theta_0 \in [0, 2\pi]$ - the real constants, $I$ - the current, $g_b(x)$, $g_c(x)$, $g_d(x)$ - given real functions and $< p >= \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta_0$ averaged value of $p$. The system (1) is supplemented by the initial conditions $p(t,0,\theta_0) = \exp(i\theta_0)$, $f(0,x) = f_0(x)$, and by the boundary conditions in the gyrotron cavity

$$f(t,0) = 0, \quad \frac{\partial f(t,L)}{\partial x} = -i\gamma f(t,L),$$

where $f_0(x)$ is the given complex function, $\gamma$ is a positive parameter.

**REFERENCES**

[1] J.Cepītis, H.Kalis and A. Reinfelds. Numerical investigations of single mode gyrotron equations. *Mathematical Modelling and Analysis*, **14** 2:169-178, 2009.

[1] This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund and by the grants 09.1220 and 09.1572 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SIMULATION OF FREE SURFACE FLOWS RELEVANT TO NUCLEAR REACTOR SAFETY

ALEXANDER CHURBANOV

*Nuclear Safety Institute, Russian Academy of Sciences*

52 B. Tulskaya, Moscow 115191, Russia

E-mail: achur@ibrae.ac.ru

Computational Fluid Dynamics (CFD) is increasingly being used to perform the safety analysis of nuclear reactors. In the document [1] there were analyzed and systematized some problems of nuclear power industry where using of CFD brings real benefits. One of these applications is Pressurized Thermal Shock (PTS) which arise at some accidents and is characterized by occurring multi-fluid systems.

Commercial CFD code FLUENT [2] has been used successfully in the present work to simulate some free surface flows relevant to the PTS problem. Moreover, a cross-verification with open source code OpenFOAM [3] designed for computational continuum mechanics has been performed, too. Two validation cases have been studied numerically here on the basis of VOF method.

First, a water jet impinging on an inclined flat plate in air environment has been predicted and compared with measurements. This validation case VAL02 has been proposed in the ECORA Project [4] for PTS problems and is based on experimental-numerical work [5]. The 3D steady-state turbulent flow has been predicted with various turbulence models and compared with measured pressure distribution along the plate.

Next, an unsteady problem of a dam break with a small obstacle placed in the way of the wave front has been calculated using both codes. In the present work comprehensive experiments performed for this problem in MARIN Institute [6] were employed for validation. Namely, time-histories of the pressure as well as height of the water layer at points of measuring were used for the comparison with 3D transient predictions.

The grid convergence and influence of the front reconstruction techniques have been studied for both cases. It was found that OpenFOAM provides a good enough agreement with experimental data as well as with predictions via FLUENT and can be utilized to construct more complicated CFD models to predict free surface flows.

**REFERENCES**

[1] B.L. Smith, U. Bieder, E. Graffard *et al.* *Assessment of Computational Fluid Dynamics (CFD) for Nuclear Reactor Safety Problems.* NEA/CSNI/R(2007)13, 2008.

[2] Fluent Inc. *FLUENT 6.3 User's Guide.* , 2006.

[3] OpenCFD Ltd. *OpenFOAM: The Open Source CFD Toolbox. User Guide v.1.7.0.* , 2010.

[4] M. Scheuerer, M. Heitsch, F. Menter *et al.* Evaluation of Computational Fluid Dynamic methods for reactor safety analysis (ECORA). *Nucl. Eng. and Design*, **235** :359–368, 2005.

[5] S. Kvicinsky. *Methode dAnalyse des Ecoulements 3d a Surface Libre: Application aux Turbines Pelto.* PhD Thesis, EPFL, Lausanne, 2002.

[6] K.M.T. Kleefsman, G. Fekken, A.E.P. Veldman *et al.* A Volume-of-Fluid based simulation method for wave impact problems. *J. Comput. Phys.*, **206** :363–393, 2005.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SIMULATION OF MULTILANE TRAFFIC FLOWS USING 2D MICROSCOPIC MODELS

NATALIA CHURBANOVA, ILYA FURMANOV and MARINA TRAPEZNIKOVA

*Keldysh Institute of Applied Mathematics RAS*

4 Muisskaya Square, Moscow 125047, Russia

E-mail: nata@imamod.ru

The paper deals with the further development of microscopic mathematical models for the simulation of vehicular traffic flows taking into account the real road geometry, i.e. the multilane structure, the presence of crossroads etc. High performance of modern supercomputers allows implementation of microscopic models with the detailed description of each vehicle behaviour at large-scale computations. The proposed model is based on the cellular automata theory. The generalization of the Nagel-Schreckenberg model [1] to the multilane case is presented. Each vehicle is characterized by its own speed and destination point, for example, the side road exit or the appointed turning at traffic lights. Vehicles have the speed limitation and should provide the safe traffic. The vehicle movement is performed under special logical laws incorporating stochastic observations. Such models are rather flexible owing to the possibility of implementing any driving strategy without substantial algorithmic costs.

The model has been verified by the number of test problems [2]. Practically important road situations have been predicted. Simulation of traffic on a crossroad demonstrates the influence of traffic lights regimes on the crossroad capacity. The road capacity depending on the entries/exits presence and on the distance between them is also investigated.

This work is supported by the Russian Foundation for Basic Research (Grants 09-01-00600, 10-01-90005).

**REFERENCES**

[1] K. Nagel and M. Schreckenberg. A Cellular Automation Model for Freeway Traffic. *J. Phys. I France*, **2** :2221, 1992.

[2] B.N. Chetverushkin, N.G. Churbanova, I.R. Furmanov and M.A. Trapeznikova. 2D micro- and macroscopic models for simulation of heterogeneous traffic flows . In: *CD-Rom Proc. of the V European Conference on Computational Fluid Dynamics ECCOMAS CFD 2010, Lisbon, Portugal, 14-17 June 2010*, J. C. F. Pereira and A. Sequeira (Eds), IDMEC, Lisbon, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON STABILITY OF FINITE DIFFERENCE SCHEMES FOR PARABOLIC PROBLEMS WITH NONLOCAL BOUNDARY CONDITIONS

RAIMONDAS ČIEGIS

*Vilnius Gediminas Technical University*

Saulėtekio al. 11, LT-10223 Vilnius, Lithuania

E-mail: rc@vgtu.lt

In this paper the one dimensional parabolic equation with integral nonlocal boundary conditions

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k(x)\frac{\partial u}{\partial x}\right) - q(x)u(x) + f(x), \quad 0 < x < 1, \ \ t > 0, \tag{1}$$

$$u(0,t) = \mu_0, \quad au(1,t) + b\int_0^1 \gamma(x)u(x,t)\,dx = g(t), \ \ t > 0, \tag{2}$$

$$u(x,0) = u_0(x), \quad 0 \le x \le 1. \tag{3}$$

is approximated by the implicit Euler finite difference scheme

$$\partial_{\bar{t}} U_j^n = \partial_{\bar{x}}\left(k_{j+1/2}\partial_x U_j^n\right) - q_j U_j^n + f_j^n, \quad x_j \in \omega_h, \ \ n > 0, \tag{4}$$

$$U_0^n = \mu_0^n, \quad aU_J^n + bS_h U^n = g^n, \quad n > 0, \tag{5}$$

$$U_j^0 = u_{0j}, \quad x_j \in \bar{\omega}_h. \tag{6}$$

Stability analysis is done in the maximum norm and it is proved that the radius of the stability region and the stiffness of the discrete scheme depends on the signs of coefficients $a, b$ in the nonlocal boundary condition.

In the case of a plain integral boundary condition ($a = 0$), the conditional convergence rate is proved and the regularization relation between discrete time and space steps is proposed. The accuracy of the obtained estimates is illustrated by results of numerical experiments. Preliminary results are presented in our paper [1].

**REFERENCES**

[1] R. Čiegis, N. Tumanova. Numerical solution of parabolic problems with nonlocal boundary conditions. *Numerical Functional Analysis and Optimization*, **31** (12):1318–1329, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# BOOTSTRAP METHODS FOR STRUCTURAL RELATIONSHIP MODELS

JURIS CIELĒNS and JĀNIS VALEINIS

*Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: juris.cielens@inbox.lv, valeinis@gmx.net

Hypothesis testing is one of the classical topics in mathematical statistics. In this work we are interested to check the goodness-of-fit of some structural relationship model, such as the classical location or location-scale models (for general notion of the structural relationship model see [1], [2]). As an alternative to the empirical likelihood method used in [3], we construct the probability-probability plot empirical process based on the structural relationship model. We derive the asymptotical limiting distribution and show how to check the hypothesis graphically by adding the simultaneous confidence bands to the probability-probability plot.

The asymptotic limiting distribution involves some nuisance parameters, the unknown distribution or density functions. Therefore to apply this test practically, one has to use some resampling methods. It appears that for the Kolmogorov-Smirnov type statistics the usual nonparametric bootstrap is not appropriate. For this reason recently the smoothed bootstrap was used in ROC curve setting (see [5]). By simulation study we compare the asymptotic behavior of our test by Monte Carlo simulations, nonparametric and smoothed bootstrap methods.

**REFERENCES**

[1] G. Freitag. *Validierung von Modellen in der Überlebenszeitanalyse*. Phd thesis, University of Bochum, Bochum, 2000.

[2] G. Freitag, A. Munk. On Hadamard diferentiability in k-sample semiparametric models - with applications to the assessment of structural relationships. *Journal of Multivariate Analysis*, **94** 1:(123-158), 2005.

[3] J. Valeinis. *Confidence bands for structural relationship models*. Phd thesis, Goerg-August-University of Goettingen, Goettingen, 2007.

[4] J. Beirlant, P. Deheuvels. On the approximation of P-P and Q-Q Plot Processes by Brownian Bridges. *Statistics & Probability Letters*, **9** (3):241-251, 1990.

[5] L. Horwáth. Confidence bands for ROC curves. *Journal of Statistical Planning and Inference*, **138** (6):1894-1904, 2008.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON THE STABILITY OF DIFFERENCE SCHEMES FOR PARABOLIC EQUATION WITH INTEGRAL CONDITIONS

R. ČIUPAILA[1], Ž. JESEVIČIŪTĖ[2], T. MEŠKAUSKAS[3] and M. SAPAGOVAS[4]

[1] *Vilnius Gediminas Technical University,* [2] *Lithuanian university of health sciences, Vytautas Magnus University,* [3] *Vilnius University, Departament of Mathematics and Informatics,* [4] *Vilnius University, Institute of Mathematics and Informatics*

[1]Saulėtekio al. 11, LT-10223, Vilnius, Lithuania, [2]Eiveniu 4, LT-50009, Kaunas, Lithuania, [3]Naugarduko 24, LT-2006, Vilnius, Lithuania, [4]Akademijos 4, LT-08663, Vilnius,Lithuania

E-mail: regimantas.ciupaila@mruni.lt, zivile.je@gmail.com

E-mail: tadas.meskauskas@mif.vu.lt, mifodijus.sapagovas@mii.vu.lt

We investigate the parabolic differential equation subject to nonlocal integral conditions with variable weight functions

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f(x,t), \quad 0 < x < 1,\ 0 \leqslant t \leqslant T,$$
$$u(0,t) = \int_0^1 \alpha(x) u(x,t) \mathrm{d}x + \mu_1(t),$$
$$u(1,t) = \int_0^1 \beta(x) u(x,t) \mathrm{d}x + \mu_2(t),$$
$$u(x,0) = \varphi(x).$$

Various difference schemes for such problem, including an accuracy $O(\tau^2 + h^4)$, were considered in articles [1; 2], where the influence of differential equation approximation methods for the stability of difference schemes were analyzed.

In our report we complement these studies and investigate the roles of weight functions $\alpha(x)$ and $\beta(x)$ for stability of difference schemes. We use the same methodology for investigation of stability analysis like in articles [3; 4].

The numerical experiments, solved in the articles [1; 2], are presented in the new aspect.

**REFERENCES**

[1] D. Dehghan. Efficient techniques for the second-order parabolic equation subject to nonlocal specifications. *Appl. Math. Comput.*, **51** (1):39–62, 2005.

[2] J. Martin-Vaquero, J. Vigo-Aguiar. On the numerical solution of the heat conduction equations subject to nonlocal conditions. *Appl. Numer. Math.*, **59** :2507-2514, 2009.

[3] M. Sapagovas. On the stability of the finite-difference schemes for nonlocal parabolic boundary-value problems. *Lithuanian Mathematical Journal*, **48** (3):339–356, 2008.

[4] Ž. Jesevičiūtė, M. Sapagovas. On the stability of the finite-difference schemes for parabolic equations subject to integral conditions with applications for thermoelasticity. *Computational Methods in Applied Mathematics*, **8** (4):360 – 373, 2008.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ALGEBRAICALLY STABLE TWO-STEP RUNGE-KUTTA METHODS FOR ORDINARY DIFFERENTIAL EQUATIONS

RAFFAELE D'AMBROSIO

*Department of Mathematics, University of Salerno*

via ponte don Melillo, 84084 Fisciano (Sa), Italy

E-mail: rdambrosio@unisa.it

We investigate algebraic stability of two-step Runge-Kutta methods [2] for ordinary differential equations using the criterion proposed by Hewitt and Hill [1] for general linear methods. This criterion is based on suitable transformations on the coefficient matrices of the methods under consideration, in such a way that the G-matrix of algebraically stable formulae is the identity matrix. This gives a remarkable improvement, since the determination of the G-matrix is, in general, a nontrivial task. Examples of algebraically stable two-step Runge-Kutta methods possessing the above feature are presented. This work is in collaboration with Zdzislaw Jackiewicz (Arizona State University), Beatrice Paternoster (University of Salerno) and Dajana Conte (University of Salerno).

**REFERENCES**

[1] L. L. Hewitt, A. T. Hill. Algebraically stable diagonally implicit general linear methods. *Appl. Numer. Math.*, **60** (6):629–636, 2010.

[2] Z. Jackiewicz. *General linear methods for ordinary differential equations.* John Wiley & Sons, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON CONCEPTS OF THE SOLUTION FOR MATRIX GAMES WITH FUZZY PAYOFFS[1]

DIANA DANCE

*Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: diana.dance@gmail.com

The classical game theory assumes that all data of a game are known by players. However, in real game situations often the players are not able to evaluate exactly some data of the game. It means that the certainty assumption is not realistic in many occasions. This lack of precision may be modeled by different ways and a fuzzy approach is one of them.

In this talk we deal with non cooperative two-person zero-sum games with fuzzy payoffs. Namely, we consider matrix games where each component of the payoff matrix is a general fuzzy number, i.e. not restricted to belong to any particular family. The earliest study of matrix games with fuzzy payoffs given with triangular fuzzy numbers is due to L. Campos [1]. The main difficulty that appears in the study of these games is the comparison between the payoff values associated to the strategies of the players because these payoffs are fuzzy quantities.

We provide a method to analyze these games finding equivalent linear programming problems with parameters whose solutions give the solutions of the games. We also give the formal definition of the value of a fuzzy payoff matrix game and propose a natural way to find it. This solution concept shares some of the most important properties listed in all approaches to this subject in the literature (see e.g. [2] and [3]).

Finally, we focus on several numerical examples to illustrate utility of our approach. We compare the values obtained by solving the corresponding parametric linear programming problem with the results obtained applying the most important models introduced by L. Campos, D.F. Li, C.R. Bector and R.R. Yager, T. Maeda, S.T. Liu and C. Kao, J.J. Buckley and L.J. Jowers, L. Xu.

**REFERENCES**

[1] L. Campos. Fuzzy linear programming models to solve fuzzy matrix games. *Fuzzy Sets and Systems*, **32** (3):275-289, 1989.

[2] C.R. Bector, S. Chandra. *Fuzzy mathematical programming and fuzzy matrix games*. Springer, Berlin, 2005.

[3] M. Larbani. Non cooperative fuzzy games in normal form: A survey. *Fuzzy Sets and Systems*, **160** (22):3184-3210, 2009.

[1]This work is supported by the project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ESTIMATION OF LONG MEMORY PARAMETER AND ITS CONFIDENCE INTERVALS

IEVA DASMANE and JĀNIS VALEINIS

*Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: ieva.dasmane@inbox.lv, valeinis@gmx.net

Long memory stationary processes have been analysed extensively in the last decades (see, for example, [1]). Although economists widely use the so called short-range dependent models such as ARIMA models, which have a property that the correlation functions decrease exponentially fast, long range dependence has been observed frequently nowadays. It started from the famous measurements of the Nile River taken by Hurst in the 1950s, when the observed correlations had much slower decay as usual.

There are plenty of methods in the literature how to estimate the long-memory parameter in stationary time series. We analyze by simulation study the recent paper [2], where Fourier-based and wavelet-based estimations were compared. As usual in statistics not only estimator itself, but sometimes the confidence intervals or statistical tests are preferable. Therefore we also combine our analysis by constructing confidence intervals for long-memory parameter based on the recent paper [4]. More specifically, we compare the confidence intervals for the wavelet estimator introduced by Abry and Veitch [3] with the confidence intervals based on some resampling methods analysed recently [4].

**REFERENCES**

[1] P. Doukhanand, G. Oppenheim and M. S. Taqqu. *Theory and applications of long-range dependence*. Birkhäuser Boston, 2003.

[2] G. Fay, E. Moulines, F. Roueff, M. S. Taqqu. Estimators of long-memory: Fourier versus wavelets. *Journal of Econometrics*, **151** (2):159–177, 2009.

[3] P. Abry and D. Veitch. Wavelet analysis of long-range-dependent traffic. *Information Theory, IEEE Transactions on*, **44** (1):2–15, 1998.

[4] P. L. Conti, L. De Giovanni, S. A. Stoev and M. S. Taqqu. Confidence intervals for the long memory parameter based on wavelets and resampling. *Statistica Sinica*, **18** (2):559–579, 2008.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# APPLICATION OF TRAVELING WAVE MODEL FOR NUMERICAL ANALYSIS OF GENERATION DYNAMICS PECULARITIES OF Q-SWITCHED SOLID-STATE LASERS

ALEKSANDR DEMENTJEV, IGOR DIOMIN, NERIJUS SLAVINSKIS[1]
RAIMONDAS ČIEGIS and INGA LAUKAITYTĖ[2]

[1] *Center for Physical Sciences and Technology (Institute of Physics)*
[2] *Vilnius Gediminas Technical University*

[1] Savanoriu pr. 231, LT-02300, Vilnius, Lithuania
[2] Sauletekio av. 11, LT-10223, Vilnius, Lithuania

E-mail: [1] aldement@ktl.mii.lt
[2] rc, ing@fm.vgtu.lt

Traveling wave model (TWM) is rarely used for the modeling of the Q-switched solid-state lasers (SSL) [1]. Sometimes it is only used for the theoretical analysis of fiber lasers [2] and laser amplifiers [3]. We present the analysis of the different regimes of the generation of Nd:YAG lasers (gain switching, active and passive Q-switching, cavity dumping) by using TWM [4], which takes into account for the possibility of direct pumping into emitting level $^4F_{3/2}$ and/or into the highly-absorbing $^4F_{3/2}$ level under the end-pumping by traveling CW or modulated pulsed pump waves, the possibility of the laser emitting from upper working level $^4F_{3/2}$ not only into the $^4I_{11/2}$ level (four-level scheme), but also into the $^4I_{11/2}$ level (quasy-three-level scheme). The Stark splitting of the upper and lower lasing levels into sublevels and their thermal Boltzmann populations are taken into consideration, as many other parameters (energy transfer up-conversion, amplified spontaneous emission, excited state absorbtion etc.) of active element (AE) and saturable absorber (SA).

The numerical simulations allowed to study the pecularities of the generation dynamics in different generation regimes, the influence of above mentioned AE and SA characteristics on generated pulse parameters such as pulse width, pulse repetition rate and pulse energy, compare the effectiveness of different pumping and lasing schemes etc. The used model allowed also to investigate the influence of seed pulse shape on the efficiency of their amplification in regenerative regime and to study the changes of their shapes during amplification.

It is shown that the developed TWM can be used successfully not only for description of Nd doped four-level lasers, but also for other quasy-three-level lasers.

**REFERENCES**

[1] Solid-State Lasers and Applications. Ed. A. Sennaroglu. *CRC Press, London*, 2007.

[2] J.-N. Maran, P. Besnard, S.LaRochelle. Theoretical analysis of a pulsed regime observed with a frequency-shifted-feedback fiber laser. *J. Opt. Soc. Am. B*, **23** (7): 1302-1311, 2006.

[3] R. Buzelis, A. Dement'ev, J. Kosenko, E. Murauskas, R. Vaicekauskas, and F.Ivanauskas. Amplification efficiency and quality alteration of short pulses amplified in the Nd:YAG amplifier in the saturation mode. *Lithuanian Phys. J.*, **38** (4): 289-301, 1998.

[4] A. S. Dement'ev, R. Čiegis, I. Laukaitytė and N. Slavinskis. Numerical analysis of laser diode longitudinally pumped solid-state laser generation dynamics using traveling wave model. *MMA*, **15** (1): 23-38, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# PERFORMANCE COMPARISON AND IMPROVEMENT OF METHODS FOR TREE SPECIES CLASSIFICATION IN MULTISPECTRAL IMAGES

ROMANS DINULS and AIVARS LORENCS

*Institute of Electronics and Computer Science*

Dzērbenes iela 14, Rīga, LV-1006, Latvia

E-mail: public.romans@gmail.com, lorencs@edi.lv

Hyperspectral and multispectral imaging is a modern tool for solving different tasks [1] including forest inventory. One of the major inventory tasks is classification of tree species.

Various methods are developed for tree species classification [2, 3, 4]. However, semiautomatic approaches that rely on prior information about trees from different species are still more practicable and precise [5]. Priory known trees are used to form design sets (DS) of the classification algorithms (CA). We compared some of these CA in terms of accuracy and succeeded in elaborating more precise and stable CA. Number of species of interest was 5.

We used CA of individual trees, that we call: $k$-nearest neighborhood classifier using weighed Euclidean metric, Bayes classifier using single pixels, Bayes classifier using mean values of pixel vectors related with one tree, Two-stage Bayes classifier (BP2), Two-stage Bayes classifier with filter for DS (BP2F), Robust two-stage Bayes classifier with filter for DS (RBP2F).

We used multispectral image of forest area in Latvia together with field data provided locations and species of about 270 trees in this area. We compared first four CA using DS with cardinality $c$ from 1 to 20 for each species. We randomly selected 100 DS for each $c$ and classified data using mentioned CA. For comparison we depicted mean, minimum and maximum correct classification percentage (excluding trees in DS). We found BP2 to be the most suitable of these CA and cardinality $c = 5$ to be the smallest suitable. Then we improved BP2 using heuristic and statistical restrictions for DS (BP2F). Finally we increased robustness of BP2F by randomly excluding trees from DS and summarizing results of 100 classifications of these reduced DS as the result of one classification (RBP2F). We performed 400 simulations of the RBP2F with $c = 5$, obtaining mean percentage of correctly classified trees 96.8% with sample standard deviation 1.3%, and no result worse than 91.7%.

## REFERENCES

[1] H. F. Grahn, P. Geladi. *Techniques and Applications of Hyperspectral Image Analysis*. John Wiley & Sons, 2007. 368 p.

[2] D. G. Leckie, S. Tinis, T. Nelson, C. Burnett, F. A. Gougeon, E. Cloney, D. Paradine. Issues in species classification of trees in old growth conifer stands. *Can. J. Remote Sensing*, **31** (2):175–190, 2005.

[3] F. Melgani, L. Bruzzone. Classification of hyperspectral remote sensing images with support vector machines. *IEEE Transactions on Geoscience and Remote Sensing*, **42** (8):1778–1790, 2004.

[4] J. N. Heinzel, H. Weinacker, B. Koch. Full automatic detection of tree species based on delineated single tree crowns - a data fusion approach for airborne laser scanning data and aerial photographs. In: *Proc. of SilviLaser 2008*, 2008. Edinburgh., 2008, 76 – 85. (in Russian)

[5] M. M. Dundar, D. Landgrebe. Toward an Optimal Supervised Classifier for the Analysis of Hyperspectral Data. *IEEE Transactions on Geoscience and Remote Sensing*, **42** (1):271–277, 2004.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# NON-MONOTONE CONVERGENCE SCHEMES

MARIA DOBKEVICH

*University of Daugavpils*

Parādes iela 1, Daugavpils, LV-5400, Latvia

E-mail: dobkevica@df.rtu.lv

We consider the second order BVP

$$x'' = f(t, x), \quad x'(0) = x'(T) = 0$$

provided thet the exist $\alpha$ and $\beta$ (lower and upper functions) such that:

$$\beta'(0) < 0 < \alpha'(0), \quad \alpha'(T) < 0 < \beta'(T).$$

We consider monotone and non-monotone approximations of solutions to the Neumann problem. The results and examples are provided.

**REFERENCES**

[1] C.De Coster, P. Habets *Two-point boundary value problems: lower and upper solutions.* Elsevier, 2006.

[2] W. Kelley, A. Peterson *The Theory of Differential Equations: Classical and Qualitative.* Pearson Education, New Jersy, 2004.

[3] L.K. Jackson, K.W. Schrader. Comparison Theorems for Nonlinear Differential Eguations. *Journal of Differential Equations*, **3**: 248–255, 1967.

[4] H. W. Knobloch. Comparison Theorems for Nonlinear Second Order Differential Equations. *Journal of Differential Equations*, **1**: 1–26, 1965.

[5] H. W. Knobloch. Second Order Differential Inequalities and Nonlinear Boundary Value Problem. *Journal of Differential Equations* , **4**: 55 – 71, 1969.

[6] Lynn H. Erbe Nonlinear Boundary Value Problems for Second Order Differential Equations. *Journal of Differential Equations*, **7**: 459 – 172, 1970.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# MOLECULAR MODELING AS A DISCRETE DYNAMICAL SYSTEM[1]

VITA DUKA[1], INESE BULA[1,2] and INTA LIEPIŅA[3]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

[3] *Latvian Institute of Organic Synthesis*

Aizkraukles iela 21, Rīga, LV-1006, Latvia

E-mail: `vita.duka@gmail.com, ibula@lanet.lv, inta@osi.lv`

Amyloids are insoluble and fibrous proteins which arise after unfolding and misfolding of soluble proteins. Abnormal accumulation of amyloids in organs may lead to amyloidosis. Amyloidosis is extracellular protein deposits that form fibrils either in the brain or other organs (Alzheimer disease). We study the formation of amyloid $\beta$-structure of the amyloid beta protein fragment 25-35 comprising 155 atoms in a molecule, by means of a beta-sheet stack system comprising 5361 atoms, from that 930 protein atoms [1]. The high cost of evaluating forces in molecular dynamics makes it necessary to use approximations. The Verlet algorithm [3] is the method of choice in molecular dynamics simulations. Schematically, the velocity Verlet algorithm is as follows.

Given $x_k$ (position) and $v_k$ (velocity) and an expression for $F(x)$ (force)

Step 1: calculate $x_{k+1} = x_k + hv_k + \frac{h^2}{2m} f(x_k)$

Step 2: evaluate $F(x_{k+1})$

Step 3: calculate $v_{k+1} = v_k + \frac{h}{2m}(F(x_k) + F(x_{k+1}))$

Now all quantities for the new step, $k+1$, have been found, go back to step 1.

The Verlet algorithm can easily be generalized to higher dimensions and many atoms. This algorithm has to be executed as many times as many iterations we would like. Besides, in any step we have to recalculate the force function $F$ value, as all atoms are moving and changing the energy interacting on every single atom. The investigation of protein molecules is time and computer recourses consuming process. Generally one of the bigest problem is force recalculation (see, for example, [2]). We would like to particularize by approximazation the force function and solve the acquired task as the system of difference equations (or discrete dynamical system). We investigate the global stability character of solutions of such system of difference equations.

**REFERENCES**

[1] V.Duka, I.Bestel, C.Czaplewsky, A.Liwo, I.Liepina. Molecular modeling of single and multiple beta-sheets of Amyloid beta protein 25 - 35. In: *Proceedings of the 31st European Peptide Symposium*, M.Lebl, M.Meldal, K.Jensen, T.Hoeg-Jensen (Editors), European Peptide Society, 2010, 572-573.

[2] R.D.Skeel, D.J.Hardy, J.C.Phillips. Correcting mesh-based force calculations to conserve both energy and momentum in molecular dynamics simulations. *Journal of Computational Physics*, **225** 1-5, 2007.

[3] L.Verlet. Computer experiments" on classical fluids. I. Thermodynamical properties of Lennard-Jones molecules. *Physical Review D*, **159** 98-103, 1967.

[1] This work was partially supported by the Latvian Council of Science research project 09.1220

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# MARKOV MODELS IN THE ANALYSIS OF FREQUENT PATTERN SEQUENCES

GINTAUTAS DZEMYDA and JULIJA PRAGARAUSKAITE

*Institute of Mathematics and Informatics, Vilnius University*

Akademijos g. 4, LT-08663 Vilnius

E-mail: julija.pragarauskaite@mii.vu.lt; gintautas.dzemyda@mii.vu.lt

Frequent sequence mining in large volume databases is important in many areas, e.g. biological, climate, financial databases. There is a number of exact methods proposed for the mining of frequent sequences, e.g. GSP, SPADE, SPAM, PrefixSpan, FreeSpan, etc. (see [1] and references therein). Exact frequent sequence mining algorithms usually read the entire database many times, and if the database is large enough, then frequent sequence mining is time consuming and expensive task.

Although there are efficient methods for mining precise and complete set of frequent patterns, approximate frequent patterns could also be used in many applications where the estimated error is acceptable, e.g. website visits database or product sales database. Approximate probabilistic algorithms are faster than exact algorithms. For example, on case of random sampling, instead of doing multiple passes over the initial database, they analyze much shorter initial database sample formed in a specific way. Moreover, in order to save the computation time, the models having Markov structure could be used. Based on the analysis results, the statistical conclusions could be made about frequent sequences in the initial database (see, e.g. [2, 3]). The errors made by the probabilistic algorithms could be estimated using statistical methods.

In this paper, we discuss approximate probabilistic models based on random sampling of the initial database and models having Markov structure. Probabilistic model makes statistical conclusions about the frequent sequences in the initial database. The errors made by the probabilistic algorithm could be estimated using statistical methods. When the initial database random sample size is increased, the error probability decreases.

## REFERENCES

[1] J. Han, H. Cheng, D. Xin, X. Yan. Frequent pattern mining: Current status and future directions. *Data Mining and Knowledge Discovery*. Springer, Volume 15, 55–86, 2007.

[2] M. El-Sayed, C. Ruiz, E. A. Rundensteiner. FS-Miner: Efficient and Incremental Mining of Frequent Sequence Patterns in Web logs. In: *Proceedings of the 6th annual ACM international workshop on Web information and data management*, WIDM '04, 2004.

[3] G. Dzemyda, R. Tumasonis. The probabilistic algorithm for mining frequent sequences. In: *Proc. of ADBIS04, 8th East-European Conf. on Advances in Databases and Information Systems*, 2004, http://www.sztaki.hu/conferences/ADBIS/8-Tumasonis.pdf.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# THE EFFECT OF FLOW CURVATURE ON LINEAR AND WEAKLY NONLINEAR INSTABILITY OF SHALLOW MIXING LAYERS

IRINA EGLITE and ANDREI KOLYSHKIN

*Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

E-mail: irina.eglite@gmail.com, akoliskins@rbs.lv

Three types of shallow flows are widespread in nature and engineering: wakes, mixing layers and jets. Linear stability analysis of shallow wake flows is performed in [1], where weakly nonlinear evolution equation (the complex Ginzburg-Landau equation) for the most unstable mode is derived. Results of linear and nonlinear modeling of shallow wakes are presented in [2].

In the present paper we analyze the effect of small curvature on the linear and weakly nonlinear instability of shallow mixing layers. Linear stability analysis is performed under the rigid-lid assumption. It is shown that the linear stability equation is the modified Rayleigh equation. The problem is solved numerically by means of a collocation method based on Chebyshev polynomials. Results of numerical calculations show that curvature has stabilizing effect on a stably curved mixing layer and destabilizes the flow for the case of an unstably curved mixing layer.

Method of multiple scales is used to derive an amplitude evolution equation for the most unstable mode if the bed-friction number is slightly smaller than the critical value. The coefficients of the amplitude equation (the complex Ginzburg-Landau equation) are calculated in closed form and are expressed in terms of the linear stability characteristics of the flow.

**REFERENCES**

[1] A.A. Kolyshkin, and M.S. Ghidaoui. Stability analysis of shallow wake flows. *J. Fluid. Mech.*, **494:** 355–377, 2003.

[2] M.S. Ghidaoui, A.A. Kolyshkin, J.H. Liang, F.C. Chan, Q. Li, and K. Xu. Linear and nonlinear analysis of shallow wakes. *J. Fluid. Mech.*, **548:** 309–340, 2006.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON LATTICES OF L-ROUGH SETS GENERATED BY L-RELATIONS[1]

ALEKSANDRS EĻKINS and ALEKSANDRS ŠOSTAKS

*Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: aleksandrs.elkins@gmail.com, aleksandrs.sostaks@lu.lv

**L-relations.** Recall that a cl-monoid is a tuple $(L, \leq, \wedge, \vee, *)$ where $(L, \leq, \wedge, \vee)$ is a complete lattice with the smallest and the largest elements $0_L$ and $1_L$ respectively, and $* : L \times L$ is a commutative associative monotone operation on $L$ which distributes over arbitrary joins, and $1_L * \alpha = \alpha$ for every $\alpha \in L$. By an $L$-relation on a set $X$ we mean an arbitrary mapping $\rho : X \times X \to L$. An $L$-relation $\rho$ is called (1) reflexive if $\rho(x, x) = 1$, (2) symmetric if $\rho(x, y) = \rho(y, x)$ and (3) transitive if $\rho(x, y) * \rho(y, z) \leq \rho(x, z)$ for all $x, y, z \in X$. There is a further binary operation - implication $\mapsto$ on a cl-monoid defined by the equality $\alpha \mapsto \beta = \bigvee\{\gamma \mid \gamma * \alpha \leq \beta\}$.

**Remark:** In case L=[0,1] and $* = \wedge$, L-relations under the name of a fuzzy relation, were first introduced by L.A. Zadeh [1] and later studied by different authors. Obviously in case when L=$\{0, 1\}$ the concept of an L-relation turns into the usual, that is crisp, concept of relation.

**L-rough sets generated by L-relations.** Given $A \in L^X$ and a reflexive, symmetric and transitive $L$-relation $\rho : X \times X \to L$ we construct an $L$-rough set $(l_\rho(A), u_\rho(A))$ where $l_\rho \colon L^X \to L^X$ and $u_\rho \colon L^X \to L^X$ are defined by the equalities $l_\rho(A)(x) = \inf_{x' \in X}(\rho(x, x') \mapsto A(x'))$ and $u_\rho(A)(x) = \sup_{x' \in X}(\rho(x, x') * A(x'))$ respectively. The pair of mappings $(l_\rho, u_\rho)$ will be referred to as an approximate system genarated by $L$-relation $\rho$.

**Remark:** In case $L = \{0, 1\}$ an $L$-rough set $(l_\rho(A), u_\rho(A))$ turns obviously into the rough set as it defined in the fundamental Z. Pawlak's work [2] and later studied by different authors. In case $L = [0, 1]$ and $* = \wedge$ the concept of an $L$-rough set under the name of a fuzzy rough set was introduced in the paper [3]. Finally the concept of an $L$-rough set as it is defined here was described in the paper [4], Section 8.2, by means of the so called approximate systems

**Lattice of L-rough sets.** Given two $L$-relations $\rho$ and $\sigma$ on $X$ we write $(l_\rho, u_\rho) \preceq (l_\sigma, u_\sigma)$ iff $l_\rho(A) \geq l_\sigma(A)$ and $u_\rho(A) \leq u_\sigma(A)$ for every $A \in L^X$. We show that $\rho \leq \sigma \Longrightarrow (l_\rho, u_\rho) \preceq (l_\sigma, u_\sigma)$. This allows to consider the family of all systems $(l_\rho, u_\rho)$, where $\rho$ is an $L$-relation on $X$, as a lattice. We study the properties of this lattice. In particular, we show that

$l_{\cap_{i\in I}\rho_i}(A)(x) \geq \bigwedge_{i\in I} l_{\rho_i}(A)(x)$, $l_{\cup_{i\in I}\rho_i}(A)(x) = \bigwedge_{i\in I} l_{\rho_i}(A)(x) \quad \forall A \in L^X, \ \forall x \in X$;

$u_{\cap_{i\in I}\rho_i}(A)(x) \leq \bigwedge_{i\in I} u_{\rho_i}(A)(x)$, $u_{\cup_{i\in I}\rho_i}(A)(x) = \bigvee_{i\in I} u_{\rho_i}(A)(x) \quad \forall A \in L^X, \ \forall x \in X$.

## REFERENCES

[1] L. A. Zadeh. Fuzzy sets. *Information and Control*, **8** No. 3, pp. 338-353, 1965.

[2] Z. Pawlak. *Rough Sets. Theoretical Aspects of Reasoning about Data.* Kluwer Acad. Publ., Boston, 1991.

[3] D. Dubois, H. Prade. Rough fuzzy sets and fuzzy rough sets. *International journal of general systems*, **17** Issue 2-3, pp. 191-209, 1990.

[4] A.Šostak. Towards the theory of M-approximate systems: Fundamentals and examples. *Fuzzy Sets and Systems*, **16** pp. 2440-2461, 2010.

[1]This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and 2009/0138/1DP/1.1.2.1.2/09/IPIA/VIAA/004 of the European Social Fund.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SIMULATION OF INTERACTING ELECTROMAGNETIC SOLITARY WAVES

D. FUNARO* and E. KASHDAN†

*Dipartimento di Matematica, Università di Modena e Reggio Emilia
e-mail: funaro@unimo.it, Web page: http://cdm.unimo.it/home/matematica/funaro.daniele
†Department of Applied Mathematics, Tel Aviv University
e-mail: ekashdan@post.tau.ac.il, Web page: http://www.math.tau.ac.il/∼ ekashdan

The scope of this research is to investigate, from the numerical viewpoint, physical phenomena arising as result of interacting electromagnetic solitary waves. The model equations introduced in [1], come from the combination of Maxwell's equations in empty space with the inviscid Euler's equation. The computational engine used for simulations is based on the high-order accurate parallel finite-difference solver ERWIN (see [2]). As the model equations suggest, the numerical studies show that electromagnetic waves can be treated exactly as a non-viscous fluid.

By setting $\rho = \vec{\nabla} \cdot \vec{E}$, the system of equations reads as follows:

$$\frac{\partial \vec{E}}{\partial t} = c^2 \vec{\nabla} \times \vec{B} - \rho \vec{V} \,, \qquad \frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} \,, \qquad \vec{\nabla} \cdot \vec{B} = 0 \,,$$
$$\frac{\partial p}{\partial t} = \mu \rho \vec{E} \cdot \vec{V} \,, \qquad \rho \left[ \frac{D\vec{V}}{Dt} + \mu (\vec{E} + \vec{V} \times \vec{B}) \right] = -\vec{\nabla} p \,,$$

where $\mu$ is a dimensional constant, $\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V}$ and $p$ is a sort of pressure. The last equation coincides with the Euler equation for the flow field $\vec{V}$, with a forcing term depending on the electromagnetic field. Such a term recalls the Lorentz law for a charged particle moving under the action of electric and magnetic fields.

The model allows us to study electromagnetic solitary waves not subjected to external perturbations (*free waves*). They are obtained in the special case when $\vec{E} + \vec{V} \times \vec{B} = 0$, $p = 0$ and $D\vec{V}/Dt = 0$ (i.e., the stream-lines are the straight lines). If for some external reasons, $D\vec{V}/Dt \neq 0$, then $\vec{V}$ changes direction, the rays are curving and the electromagnetic wave-fronts follow a new evolution. We can study in this way configurations obtained as a result of the interactions of two or more solitary waves, as well as the scattering of a soliton by an obstacle.

ERWIN is a parallel high-order accurate solver for multi-dimensional systems of nonlinear time-dependent PDEs [2]. It is based on the 4th order multi-step Runge-Kutta method (RK4) for integration in time and the 4th order accurate explicit finite-difference scheme for spatial discretization. The spatial parallelization is introduced through the MPI.

**REFERENCES**

[1] D. Funaro, *Electromagnetism and the structure of matter*, World Scientific, Singapore, 2008.

[2] E.Kashdan, *ERWIN – the parallel high-order accurate solver for multi-dimensional systems of nonlinear time-dependent PDEs with the visualization engine*, Tech. Report, School of Math Sciences, Tel Aviv University, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# LIMIT CYCLES OF THE FITZHUGH-NAGUMO NEURONAL MODEL

VALERY GAIKO

*National Academy of Sciences of Belarus (Minsk)*
*Max Plank Institute for Mathematics (Bonn)*

Leonid Beda Str. 6-4, Minsk 220040, Belarus

E-mail: valery.gaiko@yahoo.com

We study the classical FitzHugh-Nagumo dynamical system which models the spike dynamics in biological neurons [1]. This system can be written in a canonical form

$$\begin{aligned} \dot{x} &= (\gamma\,\delta - 1)\,y + (\gamma - a)\,x + b\,x^2 - c\,x^3, \\ \dot{y} &= x - \delta\,y. \end{aligned} \tag{1}$$

Such a cubic model was studied earlier, e. g., in [2]. However, the qualitative analysis carried out in [2] was incomplete, since the global bifurcations of multiple limit cycles could not be studied properly by means of the methods and techniques which were used earlier in the qualitative theory of dynamical systems. To complete the the qualitative analysis of system (1), we will use the Wintner-Perko termination principle for multiple limit cycles.

THEOREM 1. *Any one-parameter family of multiplicity-m limit cycles of a relatively prime polynomial system can be extended in a unique way to a maximal one-parameter family of multiplicity-m limit cycles of this system which is either open or cyclic. If it is open, then it terminates either as the parameter or the limit cycles become unbounded; or, the family terminates either at a singular point of the system, which is typically a fine focus of multiplicity m, or on its (compound) separatrix cycle, which is also typically of multiplicity m.*

Applying Theorem 1 and new geometric methods of the global bifurcation theory developed in [3], we prove, in particular, the following theorem.

THEOREM 2. *FitzHugh-Nagumo neuronal model (1) can have at most two limit cycles.*

**REFERENCES**

[1] E. M. Izhikevich. *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting.* The MIP Press, Cambridge, MA, 2007.

[2] M. Ringkvist and Y. Zhou. On the dynamical behaviour of FitzHugh-Nagumo systems: Revisited. *Nonlinear Anal.*, **71** 2667–2687, 2009.

[3] V. A. Gaiko. *Global Bifurcation Theory and Hilbert's Sixteenth Problem.* Kluwer Academic Publishers, Boston, 2003.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# MATHEMATICAL MODELING OF MAGNETIC COMPRESSOR OPERATION

M.P. GALANIN[1], M.K.KRYLOV[2], A.P. LOTOTCKII[2] and A.S. RODIN[1]

[1] *Keldysh Institute of Applied Mathematics of RAS*

Miusskaya sq., 4, 125047, Moscow, Russia

E-mail: `rals@bk.ru`

[2] *SRC RF TRINITI*

142190, Troitck, Moscow region, Russia

The magnetic compressor is considered [1]. The operation of the compressor is based on the compression of a magnetic field by a plain liner accelerated by electromagnetic forces to the velocity about 1 km/sec. The aim of the compressor is the input electrical pulse amplification. The output pulse power must exceed the input pulse power in dozens times. The magnetic compressor prototype is constructed in SRC RF TRINITI. A few experimental launchings is carried out on the prototype, but acquired information is insufficient because of the experiment fleetness and experimental conditions. The liner belt is destroyed at the end of the launching.

In order to receive more information the mathematical modeling of magnetic compressor operation is carried out . The 2D approximation corresponding to the transverse cross-section of spatial region is considered. The model of elastoplastic solid (case of arbitrary deformations [2-3]) is used for liner motion modeling. A set of calculations is executed. The comparison between numerical results and experimental results is performed. It is demonstrated that the constructed model makes it possible to obtain results differed from the experiment within limits of 5-10 %.

To optimize the output pulse the set of calculations with different parameters of the input electrical circuit and different inductor profile is carried out.

The work is done under partial financial support of RFBR (project No. 09-01 - 00151).

**REFERENCES**

[1] M.P.Galanin, A.P. Lototskii. Modelling the liner acceleration and deceleration in devices of power peaking. *Radiotekhnika and Electronika*, **50** (2):256–264, 2005.(in Russian)

[2] M.P.Galanin, A.P. Lototskii, A.S. Rodin and I.A. Sheglov. The liner motion in the transverse section of magnetic compressor. *Vestnic MGTU*, **1** (2):65–84, 2010.(in Russian)

[3] M.P.Galanin, A.P. Lototskii, A.S. Rodin . The liner motion in the various sections of the magnetic compressor. *Mathematical Modelling*, **22** (10):35–55, 2010.(in Russian)

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# LIMIT THEOREMS WITH INCREASING MODULUS FOR TWISTED $L$-FUNCTIONS OF ELLIPTIC CURVES

VIRGINIJA GARBALIAUSKIENĖ

*Šiauliai University*

P. Višinskio 19, LT-77156 Šiauliai , Lithuania

E-mail: `virginija@fm.su.lt`

Let $E$ be a non elliptic curve over the field of rational numbers, and let $L_E(s)$, $\sigma+it$, denote the $L$-function attached to $E$. We consider the value-distribution of twisted with Dirichlet character $L$-function $L_E(s,\chi)$. The function $L_E(s,\chi)$ is obtained from $L_E(s)$ by replacing the coefficients $\lambda(m)$ of the series for $L_E(s)$ by $\lambda(m)\chi(m)$, where $\chi(m)$ is a Dirichlet character modulo $q$.

Suppose that $q$ is a prime number, and

$$M_Q=\sum_{q\le Q}\sum_{\substack{\chi=\chi\,(\mathrm{mod}\,q)\\ \chi\ne\chi_0}}1,$$

where $\chi_0$ is the principal character. We prove limit theorems for probability measures

$$M_Q^{-1}\sum_{q\le Q}\sum_{\substack{\chi=\chi\,(\mathrm{mod}\,q)\\ \chi\ne\chi_0\\ |L_E(s,\chi)|\in A}}1,\quad A\in\mathcal{B}(\mathbb{R}),$$

and

$$M_Q^{-1}\sum_{q\le Q}\sum_{\substack{\chi=\chi\,(\mathrm{mod}\,q)\\ \chi\ne\chi_0\\ \exp\{i\arg L_E(s,\chi)\}\in A}}1,\quad A\in\mathcal{B}(\gamma),$$

as $Q\to\infty$. Here $\mathcal{B}(S)$ denotes the class of Borel sets of the space $S$, and $\gamma$ is the unit circle on the complex plane.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ZEROS OF THE ESTERMANN ZETA-FUNCTION

RAMŪNAS GARUNKŠTIS

*Vilnius university*

Naugarduko 24, Vilnius, Lithuania

E-mail: ramunas.garunkstis@mif.vu.lt

We investigate the zeros of the Estermann zeta-function

$$E\left(s;\frac{k}{\ell},\alpha\right)=\sum_{n=1}^{\infty}\frac{\sigma_\alpha(n)}{n^s}\exp\left(2\pi in\frac{k}{\ell}\right)$$

as a function of a complex variable $s$, where $k$ and $\ell$ are coprime integers and

$$\sigma_\alpha(n)=\sum_{d|n}d^\alpha$$

is the generalized divisor function with a fixed complex number $\alpha$. In particular, we study the question on how the zeros of $E(s;\frac{k}{\ell},\alpha)$ depend on the parameters $\frac{k}{\ell}$ and $\alpha$. It turns out that for some zeros there is a continuous dependency whereas for some other zeros there is none. This is a joint work with A. Dubickas, J. Steuding, and R. Steuding.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# A FINITE DIFFERENCE ANALYSIS FOR SOME COUPLED POROMECHANICS PROBLEMS

F.J. GASPAR, N. BOAL, F.J. LISBONA and P.N. VABISHCHEVICH

*Zaragoza University*
Department of Applied Mathematics
Pedro Cerbuna 12
50009 Zaragoza
Spain

E-mail: fjgaspar@unizar.es

This work is focused on the numerical analysis of two coupled poromechanics problems. The first considered problem is the double porosity consolidation model for fractured rock formations. The poroelastic behaviour of double-porosity rocks depends on both the flow and mechanical parameters of the fracture as well as the matrix. Therefore, this model involves a coupling among the displacements of the solid and the fluid pressure for pores and fissures. The second problem corresponds to the linear thermoporoelastic model, which involves a coupling among three fields: displacement field for solid skeleton, pore pressure field for fluid flow and temperature field for heat transfer. In both coupled poromechanics problems, standard finite difference methods exhibit non-physical oscillations. The oscillations can be overcome by locally refining the mesh especially near the draining boundary. An alternative solution is to use stabilized methods based on staggered meshes, obtaining solutions free of oscillations for any values of discretization parameters. A priori estimates in discrete energy norms are obtained and convergence results are given. Finally, some numerical experiments, which confirm the theoretical results, are shown.

This research has been funded by DGA and project MTM 2010-16917.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# HIGHER ORDER FINITE DIFFERENCE SCHEMES FOR PERIODICAL BOUNDARY CONDITIONS[1]

AIGARS GEDROICS[1] and HARIJS KALIS[1,2]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: aigors@inbox.lv, kalis@lanet.lv

For approximation of the operator $-\frac{\partial^2}{\partial x^2}$, $(x \in [0, L])$ with periodical boundary conditions finite difference expressions with the different order of approximation are investigated. We consider corresponding discrete spectral problem $Ay = \mu y$ for finite difference operator on uniform grid $x_j = jh, j = \overline{1, N}, Nh = L$ where $h$ is the grid parameter, $\mu$ is eigenvalue and $A, y$ are the circulant matrix and column-vector of $N$ order (eigenvector) with elements $y_j, j = \overline{0, N}$.

We use from two vectors $y^1$, $y^2$ following scalar product $\left[y^1, \bar{y}^2\right] = h\left(\sum_{j=1}^{N} y_j^1 \bar{y}_j^2\right)$, where $\bar{y}$ is the conjugate value of $y$. The corresponding discrete spectral problem $Ay^n = \mu_n y^n$, $n = \overline{1, N}$ with circulant matrix $A$ have following eigenvectors: $y^n = C_n^{-1}(y_1^n, y_2^n, \ldots, y_N^n)^T$, where $y_j^n = \exp(2\pi i n x_j/L)$, $j = \overline{1, N}$, $i = \sqrt{-1}$ are the components of column-vector $y^n$. The constants $C_n = \sqrt{N}$ and we have the orthonormed eigenvectors $y^n$.

We obtain the matrix $A$ and eigenvalues $\mu_n$ of matrix $A$ for different order of approximation $O(h^k), k \geq 2$:

1. $k = 2, h^2 A = [2, -1, 0, \cdots, 0, -1]$, $h^2\mu_n = 4\sin^2(\pi n/N)$,
2. $k = 4, h^2 A = [\frac{5}{2}, -\frac{4}{3}, \frac{1}{12}, 0, \cdots, 0, \frac{1}{12}, -\frac{4}{3}]$, $h^2\mu_n = 4(\sin^2(\pi n/N) + \frac{1}{3}\sin^4(\pi n/N))$,
3. $k = 6, h^2 A = [\frac{49}{18}, -\frac{3}{2}, \frac{3}{20}, -\frac{1}{90}, 0, \cdots, 0, -\frac{1}{90}, \frac{3}{20}, -\frac{3}{2}]$, $h^2\mu_n = 4(\sin^2(\pi n/N) + \frac{1}{3}\sin^4(\pi n/N) + \frac{8}{45}\sin^6(\pi n/N))$,
4. $k = 8, h^2 A = [\frac{205}{72}, -\frac{8}{5}, \frac{1}{5}, -\frac{8}{315}, \frac{1}{560}, 0, \cdots, 0, \frac{1}{560}, -\frac{8}{315}, \frac{1}{5}, -\frac{8}{5}]$,
   $h^2\mu_n = 4(\sin^2(\pi n/N) + \frac{1}{3}\sin^4(\pi n/N) + \frac{8}{45}\sin^6(\pi n/N) + \frac{4}{35}\sin^8(\pi n/N))$, etc.

Therefore the matrix $A$ can be represented in the form $A = PDP^*$, where the column of the matrix $P$ and the diagonal matrix $D$ contains $N$ orthonormed eigenvectors $y^n$ and eigenvalues $\mu_n, n = \overline{1, N}$ correspondly. From $P^*P = E$ follows that $P^{-1} = P^*$, where $E$ is the unit matrix. For solving the problems of the mathematical physics we compare these methods with the scheme with the exact spectrum, when in the matrix $D$ elements are the first $N$ eigenvalues of the continuous differential operator $\lambda_n = (\frac{2n\pi}{L})^2$.

[1] This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund and by the grant 09.1572 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# COUPLING OF EM AND HD PROBLEMS FOR SIMULATION OF INDUSTRIAL MELTING PROCESS

JURIJS GREČENKOVS[1], MIHAILS ŠČEPANSKIS[1], ANDRIS JAKOVIČS[1] and BERNARD NACKE[2]

[1] *University of Latvia, Faculty of Physics and Mathematics, Department of Physics, Laboratory for Mathematical Modelling of Environmental and Technological Processes*

Zeļļu iela 8, LV-1002, Rīga, Latvia

[2] *Leibniz University of Hanover, Institute of Electrotechnology*

Wilhelm-Busch-Str. 4, D-30167, Hannover, Germany

E-mail: jurijs.grecenkovs@gmail.com

Electromagnetic (EM) heating and melting are one of the most effective methods for conducting material processing and production (the typical equipment is the induction crucible furnace).

Considered physical problem consists of coupled EM, hydrodynamic (HD) and thermal problems. The extent of the coupling depend on material properties of the conducting liquid. The equation for EM field written in terms of complex vector potential $\mathbf{A}$ in the case of harmonic current $\mathbf{j}$ is as follows:

$$\Delta \mathbf{A} = \mu\mu_0 \left(-\mathbf{j} + \imath\omega\mathbf{A}\right), \tag{1}$$

where $\imath$ is imaginary unit, $\omega$ is radial frequency, $\mu_0$ and $\mu$ are vacuum and relative magnetic permeabilities respectively. The two-ways coupled HD and thermal problems can be described with the Navier-Stokes, continuity and heat transfer equations:

$$\begin{cases} \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\Delta\mathbf{v} - \alpha\left(T - T_0\right)\mathbf{g} + \frac{1}{\rho}\mathbf{f_{em}}; \\ \nabla\cdot\mathbf{v} = 0; \\ \frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T - \lambda\Delta T = q_{Joule}, \end{cases} \tag{2}$$

where $\rho$ is density, $\mathbf{v}$ is velocity, $p$ is pressure, $\nu$ is viscosity, $\alpha$ is heat expansion coefficient, $T$ is temperature, $T_0$ is reference temperature, $\lambda$ is thermal diffusivity, $\mathbf{g}$ is free fall acceleration, $q_{Joule}$ and $\mathbf{f_{em}}$ are Joule heat and Lorentz force densities derived from $\mathbf{A}$.

Different approaches for the simulation of the system (2) (Reynolds Averaged Navier-Stokes, Large Eddy Simulation) are coded within OpenFOAM free software libraries [1] by means of finite volume (FV) method. The objective of the present work is to create the solver block for equation (1), which can be implemented in OpenFOAM code. As fare as equation (1) is defined for complex function and the concerned industial problems have quite simple geometry, the finite difference (FD) method is consequently used for solution of equation (1). This algorithm and the additional procedure for coupling of FD and FV methods are implemented in OpenFOAM libraries.

**REFERENCES**

[1] H. Rusche. *Computational Fluid Dynamics of Dispersed Two-Phase Flows at High Phase Fractions (PhD thesis).* Imperial College of Science, Technology and Medicine, Department of Mechanical Engineering, London, 2002.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# INVOLVING FUZZY ORDERS FOR MULTI-OBJECTIVE LINEAR PROGRAMMING

OLGA GRIGORENKO

*Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga LV-1002, Latvia

E-mail: ol.grigorenko@gmail.com

In our work we observe a multi-objective linear programming problem, which can be represented as follows: $MAX\ Z$, where $Z = (z_1 \dots, z_k)$ is a vector of objectives, $z_i = \sum_{j=1}^{n} c_{ij}x_j$ where $i = 1, \dots, k$, subject to $\sum_{j=1}^{n} a_{ij}x_j \leq b_i$, $i = 1, ..., m$. That is we should find a vector $x^P = (x_1^P, \dots, x_n^P)$ which maximizes $k$ objective functions with $n$ variables, and $m$ constraints. There are many approaches to solve this problem and among them is a fuzzy approach (see e.g. [3]). In our work we propose to use fuzzy order relations in order to solve multi-objective linear programming problems in the following way:

1. We define fuzzy order relations $P_i$ which generalize the following crisp weak order relations $x \preceq_i y \Leftrightarrow z_i(x) \leq z_i(y)$, $i = 1, \dots, k$ (see [1]). Thus each fuzzy order relation describes corresponding objective function $z_i$.
2. We aggregate fuzzy orders using an aggregation function $A$ which preserves the properties of initial fuzzy orders (see [2]).

$$P(x, y) = A(P_1(x, y), \dots, P_k(x, y)).$$

   Thus the aggregated fuzzy order relation $P$ provides the information about all objective functions.
3. Further the multi-objective linear programming problem comes to the following problem:

$$\max_y \min_x P(x, y).$$

In the current work we also provide illustrative examples and study the results corresponding to Pareto optimal solution.

**REFERENCES**

[1] U. Bodenhofer. A similarity-based generalization of fuzzy orderings preserving the classical axioms. *Internat. J. Uncertain. Fuzziness Knowledge-Based Systems*, **8** (5):593–610, 2000.

[2] S. Saminger, R. Mesiar and U. Bodenhofer. Domination of aggregation operators and preservation of transitivity. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, **10** (Suppl.):11–35, 2002.

[3] H.-J. Zimmermann. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, **1** (1):45–55, 1978.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ZEROS OF THE POLYLOGARITHM FUNCTION

ANDRIUS GRIGUTIS

*Vilnius university*

Naugarduko 24, Vilnius, Lithuania

E-mail: andrius.grigutis@mif.stud.vu.lt

We investigate the zeros and trajectories of zeros of the Polylogarithm function

$$f(s,q,\alpha)=\sum_{n=0}^{\infty}\frac{q^n}{(n+\alpha)^s}$$

as a function of a complex variable $s$, where $0<q\leq 1$ and $0<\alpha\leq 1$.

In particular, we study the question on how the trajectories of zeros behave when $q$ is close to 1 and $\alpha=1$. If $q=1$ the Polylogarithm function becomes the classical Hurwitz zeta-function $\zeta(s,\alpha)=f(s,1,\alpha)$, if $q=1$ and $\alpha=1$ the Polylogarithm function becomes the well known Riemann zeta-function $\zeta(s)=f(s,1,1)$.

We prove that $f(s,q,\alpha)$ vanishes in half-plane $\sigma>1$ if $\alpha$ is transcendental. This is a joint work with R. Garunkštis.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SOLVABILITY OF BOUNDARY VALUE PROBLEMS WITH ASYMMETRIC PRINCIPAL PARTS[1]

ARMANDS GRITSANS and FELIX SADYRBAEV

*Daugavpils University*

Parādes iela 1, Daugavpils, LV-5400, Latvia

E-mail: arminge@inbox.lv

The problems of the type

$$x'' = -\lambda f(t, x^+) + \mu g(t, x^-) + h(t, x, x'), \quad x(0) = x(T) = 0$$

are considered where $x^+ = \max\{x, 0\}$, $x^- = \max\{-x, 0\}$. The existence results are provided which are based on the analysis of the spectrum of the related Fučík type problem.

**REFERENCES**

[1] F. Sadyrbaev. Multiplicity in parameter-dependent problems for ordinary differential equations. *Math. Model. Anal.*, **14** (4):503-514, 2009.

[1] This work was partially supported by the Latvian Council of Science research project 09.1220

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON REPRESENTATION FORMULA FOR SOLUTIONS OF HAMILTON - JACOBI EQUATION WITH LOWER SEMICONTINUOUS INITIAL DATA

GINTAUTAS GUDYNAS

*Klaipeda university*

H. Manto g. 84, Klaipeda, Lithuania

E-mail: gvgintaut@balticum-tv.lt

We give the representation formula for solutions of Cauchy problem

$$u_t + H(t, x, u, u_x) = 0, \tag{1}$$

$$u(0, x) = \varphi(x)$$

in domain $S_T = \{(t, x) : t > 0, x \in R^n\}$. The hamiltonian $H(t, x, u, p)$ is assumed to be strictly convex in $p$ and $\varphi(x)$ - lower semicontinuous. The unique semiconcave solution is given by

$$u(t, x) = \lim_{\xi \in R^n} \Phi(t, x, \xi),$$

where $\Phi(t, x, \xi)$ is the solution of (1) satisfied the initial condition

$$\Phi(t, x, \xi) = \begin{cases} \varphi(\xi), & x = \xi \\ +\infty, & x \neq \xi \end{cases}.$$

**REFERENCES**

[1] O. Alvarez, E. N. Barron, H. Ishii. Hopf - Lax formulas for semicontinuous data. *Indiana University Mathematics Journal*, **48** (3):993-1035, 1999.

[2] S. N. Kruzkov. Generalized solutions of the multi-dimensional nonlinear partial differential equations of first order. *II. Math. sbornik*, **77** (112):108-134, 1967.

[3] P. Cannarsa, C. Sinestrari. *Semiconcave Functions, Hamilton-Jacobi Equations and Optimal Control. Progress in Nonlinear Differential Equations and Their Applications, Vol. 58*. Birkhauser Verlag, Basel-Boston-Berlin, 2004.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# NAVIER-STOKES EQUATION IN MAGNETIC FIELD

SERGEJS HILKEVICS and GALINA HILKEVICA

*Ventspils University College*

Inženieru iela 101, Ventspils, LV-3600, Latvia

E-mail: sergejs.hilkevics@venta.lv, galina.hilkevica@venta.lv

Recent optical observations of galaxy center demonstrated the existence of bispiral flows of plasma in cosmic space, which are very similar to experimentally observed flows of conducting liquids in magnetohydrodynamic Riga dynamo experiments at the Institute of Physics in Salaspils [1; 2]. Using methods of very large base radiointeferometry it is possible receive resolution of such observations more than 1000 times higher comparing with optics observations, which makes such objects study as very interesting for 32 meters radiotelescope of Ventspils International Radioastronomy Centre. For bispiral liquid and gas flows dynamics understanding it is necessary to solve the system of magnetohydrodynamic equations, which in the case of incompressible liquid looks as [3]

$$\operatorname{div} \mathbf{H} = 0. \tag{1}$$

$$\operatorname{div} \mathbf{v} = 0. \tag{2}$$

$$\frac{\partial \mathbf{H}}{\partial t} + (\mathbf{v}\nabla\mathbf{H}) = (\mathbf{H}\nabla)\mathbf{v} + \frac{c^2}{4\pi\sigma}\Delta\mathbf{H}. \tag{3}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = \frac{1}{\rho}\nabla(\mathsf{P} + \frac{\mathsf{H}^2}{8\pi}) + \frac{1}{4\pi\rho}(\mathbf{H}\nabla)\mathbf{H} + \nu\boldsymbol{\Delta}\mathbf{v}. \tag{4}$$

The solutions of this system for different initial and border conditions provide solutions with different topology of flows, including bispiral configurations mentioned above. Results of modeling can be explained using qualitative approach based on extremal action principle.

**REFERENCES**

[1] F. Stefani, A. Gailitis and G. Gerbeth. Magnetohydrodynamic experiments on cosmic magnetic fields. *Zeitschrift fur Angewandte Mathematik und Mehanik*, 2 October, 2008.

[2] http://ipul.lv/main/?nav=RigaD. . , .

[3] D. Landau, E. Lifshits. Elektrodinamika sploshnih sred *Nauka, Moscow.* , 1982, 620.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# LINEAR/LINEAR RATIONAL SPLINE COLLOCATION FOR BOUNDARY VALUE PROBLEMS

ERGE IDEON and PEETER OJA

*Institute of Mathematics, University of Tartu*

J.Liivi 2, 50409 Tartu, Estonia

E-mail: Erge.Ideon@ut.ee, Peeter.Oja@ut.ee

We consider the boundary value problem

$$y''(x) + p(x)y'(x) + q(x)y(x) = f(x), \quad x \in (a, b),$$

$$y(a) = \alpha, \quad y(b) = \beta,$$

and suppose that it has the unique solution. We will discuss the linear/linear rational spline collocation on the uniform mesh with $x_i = a + ih$, $i = 0, \ldots, n$, $h = (b - a)/n$, as spline knots using collocation points $\xi_i = (x_{i-1} + x_i)/2$, $i = 1, \ldots, n$. The linear/linear rational spline $S$ is assumed to be of class $C^1$ and of the form

$$S(x) = a_i + \frac{c_i(x - \xi_i)}{1 + d_i(x - \xi_i)}, \quad x \in [x_{i-1}, x_i], \quad 1 + d_i(x - \xi_i) > 0, \quad i = 1, \ldots, n.$$

In addition, we require the spline to satisfy the boundary conditions $S(a) = \alpha$, $S(b) = \beta$. The collocation method leads to a nonlinear system to determine the spline coefficients $a_i$, $c_i$, $d_i$.

Note that the linear/linear rational spline being constant or strictly monotone by itself is a reasonable approximate solution only if the exact solution of the problem has the same property.

We give numerical examples which show the convergence with order $O(h^2)$.

It is known that at interpolation of functions rational splines may have advantage compared to polynomial splines [3]. The cubic and quadratic spline collocation methods on uniform mesh are both of order $O(h^2)$ but the error of either of them may be really considerably grater than the error of other one, see [2]. Let us mention that $O(h^2)$ convergence rate of quadratic spline collocation for boundary value problems is based on superconvergence property of interpolating splines in certain points [2]. The superconvergence of the linear/linear rational spline interpolation is studied in [1].

In our study we use the spline representation by spline values $S_i = S(x_i)$, $i = 0, \ldots, n$, $\bar{S}_i = S(\xi_i)$, $i = 1, \ldots, n$,

$$S(x) = \bar{S}_i + \frac{4(S_i - \bar{S}_i)(\bar{S}_i - S_{i-1})(x - \xi_i)}{h(S_i - S_{i-1}) + 2(2\bar{S}_i - S_{i-1} - S_i)(x - \xi_i)}, \quad x \in [x_{i-1}, x_i].$$

**REFERENCES**

[1] E. Ideon and P. Oja. Linear/linear rational spline interpolation. *MMA*, **15(4),** 2010, 447–455.

[2] P. Oja and A. Reitsekas. On the collocation and subregions methods with quadratic and cubic splines for the boundary value problems. *Proc. Estonian Acad. Sci. Phys. Math.*, **36(2),** 1987, 118–128.

[3] P. Oja. Low degree rational spline interpolation. *BIT*, **37(4),** 1997, 901–909.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# DETERMINATION OF THERMAL CONDUCTIVITY COEFFICIENT BY USING TEMPERATURE FIELD PROJECTION IN A SERIES ALONG BOUNDARY CONDITION DERIVATIVES

MARIJA ILTINA and ILMARS ILTINS

*Riga Technical University*

Meža iela1/4, Rīga, LV-1048, Latvia

E-mail: marijai@inbox.lv

In most cases inverse heat exchange problems are solved by using test data obtained in the mathematically possibly simpliest field. Therefore let us confine ourselves to a one-dimensional temperature field that could be written down in non-dimensional form as follows:

$$\frac{\partial t}{\partial F} = a^2\left(\frac{\partial^2 t}{\partial N^2} + \frac{k-1}{N}\frac{\partial t}{\partial N}\right), \tag{1}$$

where $F$ and $N \in [0, b]$ dimensionless time and coordinate, respectively, $a$ is the coefficient of thermal conductivity and $k = 1, 2$ or 3.

To determine $a$ with any boundary conditions, one should also know a temperature change at some point $N_1 \in [0, b)$. Solution of (1) with boundary conditions of the first kind can be represented in the following form [1]:

$$t(N, F) = \sum_{n=0}^{\infty}[P_n(N, k)t_1^{(n)}(F) + Q_n(N, k)t_2^{(n)}(F)], \tag{2}$$

where $t_1(F)$ and $t_2(F)$ are temperatures at the boundary and $Q_n = 0$ if the temperature distribution is symmetric. Functions $P_n$ and $Q_n$ for various geometries and different kinds of boundary conditions can be found in [1] and [2]. It follows from (2) that if temperatures at the boundaries $t_1(\tau)$ and $t_2(\tau)$ are measured where $\tau$ is time, then at some interior point $N_1 \in [0, b)$ the partial sums of (2) are the following:

$$t(N_1, \tau) = \sum_{n=0}^{m} R_n(N_1, \tau)b^{(2n)}a^{-n}, \tag{3}$$

where $R_n(N_1, \tau)$ are the values obtained from the test results. The expression (3) is a polynomial of degree $m$ in $b^2/a$. The problem here is that the polynomial has $m$ roots, but there is only one coefficient of thermal conductivity. The problem of interest is how valid and invalid roots can be distinguished.

**REFERENCES**

[1] A. Temkin. *Inverse methods of thermal conductivity.* Energya, Moscow, 1973.

[2] A. Temkin, I. Gerhards, and I. Iltins. The temperature field of cable insulation. *Latvian Journal of Physical and Technical Sciences*, **2:** 12–26, 2005.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# THE STABILITY OF THE DIFFERENCE SHEMES FOR PSEUDOPARABOLIC EQUATION SUBJECT TO NONLOCAL CONDITIONS (WITH APPLICATIONS TO UNDERGROUND WATER FLOW)

JUSTINA JACHIMAVIČIENĖ and MIFODIJUS SAPAGOVAS

*Vilnius University, Institute of Informatics and Mathematics*

Akademijos 4, LT-08663, Vilnius, Lithuania

E-mail: justina_jachimaviciene@yahoo.com, m.sapagovas@ktl.mii.lt

We consider the stability of difference schemes for a pseudo-parabolic equation with constant coefficients

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial}{\partial t}\Big(\frac{\partial^2 u}{\partial x^2}\Big) + f(x,t) \tag{1}$$

and nonlocal conditions, which are determined so that the differential problem could be interpreted as the underground water flow problem. Examples of this kind of nonlocal conditions are [1]:

$$\frac{\partial}{\partial t}\int_0^l u(x,t)\mathrm{d}x = \tau(t) \tag{2}$$

or

$$\frac{\partial u(0,t)}{\partial x} = \sigma(t)\frac{\partial}{\partial t}\int_0^l u(x,t)\mathrm{d}x + \varphi(t). \tag{3}$$

So the stability schemes are investigated by analyzing the structure of spectrum of the transaction matrix. Difference schemes which approximate the differential problem with an accuracy $O(\tau + h^2)$, $O(\tau^2 + h^2)$, $O(\tau^2 + h^4)$.

**REFERENCES**

[1] A.M.Nakhushev. *Equations of mathematical biology.* "Vysshaya shkola" Publishing House, Moscow, 1995. (In Russian)

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# GENERAL LINEAR METHODS WITH LARGE REGIONS OF ABSOLUTE STABILITY

ZDZISŁAW JACKIEWICZ

*School of Mathematical & Statistical Sciences*

Arizona State University, Tempe, Arizona 85287

E-mail: jackiewi@math.la.asu.edu

We describe the construction of general linear methods in Nordsieck form of order $p$ and stage order $q = p$ with large regions of absolute stability. We review the concepts of Runge-Kutta and quadratic stability and inherent Runge-Kutta and inherent quadratic stability [3] which aid in the construction of general linear methods with desirable stability properties. We also derive the representation formulas for some of the coefficient matrices of Nordsieck methods [1]. The search for these methods is based on maximizing the area of the intersection of the region of absolute stability with the negative complex plane by various optimization routines [2]. We first construct quadratic polynomials with large regions of absolute stability and then search for methods whose stability functions matches these stability polynomials. The efficient computation of coefficients of these polynomials utilizes the fast Fourier transform. Examples of methods obtained in this way are presented up to the order six. This is a joint work with A. Cardone from University of Salerno, and H. Mittelmann from Arizona State University.

**REFERENCES**

[1] A. Cardone and Z. Jackiewicz. Explicit Nordsieck methods with quadratic stability. Submitted.

[2] A. Cardone, Z. Jackiewicz and H. Mittelmann. Search for Nordsieck methods of high order with quadratic stability. Manuscript.

[3] Z. Jackiewicz. *General Linear Methods for Ordinary Differential Equations*. John Wiley & Sons, Hoboken, New Jersey, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# TWO–DIMENSIONAL PARABOLIC EQUATION WITH NONLOCAL INTEGRAL BOUNDARY CONDITIONS

KRISTINA JAKUBĖLIENĖ and MIFODIJUS SAPAGOVAS

*Institute of Informatics and Mathematics*

Akademijos 4, LT–08663, Vilnius, Lithuania

E-mail: m.sapagovas@ktl.mii.lt, gibaite@gmail.com

We consider the two–dimensional parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f(x,y,t), \quad 0 < x,y < 1 \quad 0 < t \leqslant T$$

with two clasical boundary conditions, and with two nonlocal integral boundary conditions

$$\begin{aligned}
&u(0,y,t) = \mu_1(y,t), \quad 0 < y < 1, \quad 0 < t \leqslant T,\\
&u(1,y,t) = \mu_2(y,t), \quad 0 < y < 1 \quad 0 < t \leqslant T,\\
&u(x,0,t) = \iint\limits_{\Omega} \gamma_3(x) u(\xi,\eta,t) d\xi d\eta + \mu_3(x,t) \quad 0 < x < 1 \quad 0 < t \leqslant T,\\
&u(x,1,t) = \iint\limits_{\Omega} \gamma_4(x) u(\xi,\eta,t) d\xi d\eta + \mu_4(x,t) \quad 0 < x < 1 \quad 0 < t \leqslant T
\end{aligned}$$

and initial condition

$$u(x,y,0) = \varphi(x,y) \quad 0 \leqslant x,y \leqslant 1.$$

We construct and investigate for this diferential problem the implicit alternating direction method.

**REFERENCES**

[1] R. Čiupaila, K. Jakubėlienė and M. Sapagovas. Solution of two–dimensional elliptic equation with nonlocal. *DATA–2009 Conference*, pp. 92–98, 2009.

[2] Yanping Lin, Shuzhon Xu and Hong–Ming Yin. Finite difference approximations for a class of non–local parabolic equations. *Math*, **20** pp. 147–164, 1997.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# DIFFUSION OF QUANTUM WAVES

A. J. JANAVIČIUS

*Šiauliai University*

P. Višinskio 19, LT-77156, Šiauliai, Lithuania

E-mail: AYanavy@gmail.com

In quantum mechanic, free particle motion is associated with harmonic plane wave vector $\frac{2\pi}{\lambda}$ and momentum $p = \hbar k$, $\hbar = 6.5819 \cdot 10^{-22} MeV \cdot sec$ which satisfy de Broglie relation $\lambda = \frac{h}{p}$. For example, we can suppose [1] that moving electron can emit an energetic photon, or a photon may convert into an electron-positron pair not only in the interaction processes but also in the free movement case. This microscopic disorder or virtual processes [1] we represented by the quantum diffusion equation in physical

$$\frac{\partial \Psi_J}{\partial t} = D_C \frac{\partial^2 \Psi_J}{\partial x^2}, \quad D_C = \frac{\hbar}{4m} = \frac{\omega}{2k^2}.$$

This expression of diffusion coefficient $D_C$ we can obtain from Heisenberg uncertainties for momentum and energy oscillations in physical vacuum

$$D_C = \frac{1}{2}\Delta x^2 \frac{1}{\Delta t} = \frac{\hbar}{4m}, \quad 2mc\Delta x = \hbar, \quad 2mc^2 \cdot \Delta t = \hbar.$$

Requiring that solution of quantum diffusion equation must represent some kind plain waves, we can propose physical solution in the following form

$$\Psi_J = Ae^{-i\omega t + ikx - kx}, \quad k = \sqrt{\frac{\omega}{2D_C}}, \quad k^2 = \frac{2mE}{\hbar}^2.$$

Substituting this solution into presented quantum diffusion equation we obtain

$$\omega \Psi_J = \frac{\hbar k^2}{2m}\Psi_J, \quad E\Psi_J = \frac{p^2}{2m}\Psi_J, \quad p = \hbar k$$

de Broglie equation $\lambda = \frac{h}{p}$ is satisfied. In this case, we must require the periodicity of plain wave. For this aim, we must introduce coordinate $x = n\lambda + x_\lambda$, $0 < x_\lambda < \lambda$ and quantum diffusion processes can acquire only in the region limited by $x_\lambda$ region. In this case plain wave represent some confined packet fixed in the region limited by kinetic or bound energy $E$ and the term

$$R = k^{-1} = \frac{\lambda}{2\pi} = \sqrt{\frac{2D_C}{\omega}} = \frac{\hbar}{p} = \frac{\hbar}{\sqrt{2m|E|}}.$$

**REFERENCES**

[1] G.D. Coughlan and J.E. Dodd. *The ideas of particle physics.* Cambridge University Press, Cambridge, 1991.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# THE POTENTIAL REPRESENTATION SOLUTION OF THE SCHRÖDINGER EQUATION FOR BOUND STATES IN MANY PARTICLES CASE

A. J. JANAVIČIUS, D. JURGAITIS and K. KIRILIAUSKAITĖ

*Šiauliai University*

P. Višinskio 19, LT-77156, Šiauliai, Lithuania

E-mail: AYanavy@gmail.com, pletra@cr.su.lt

The eigenfunctions in the potential representation of Schrödinger equation can be find [1] $U_\alpha(\vec{r}_i) = \prod_{i\neq j}^{N}\langle\varphi_\alpha(\vec{r}_{ij})\rangle_J U_{nL}(\vec{r}_i)$ as a product of the unperturbed one-particle wave functions $U_{nL}$ for central potential $V_0(r)$ and mean value of multiplier $\varphi_\alpha$, which depends on the perturbation potential including interaction between $N$ particles

$$\frac{d^2}{dr^2}U_\alpha - \frac{L(L+1)}{r^2}U_\alpha + C(E_\alpha - V_0(r) - V_H(r))U_\alpha = 0,$$

$$C = \frac{2m}{\hbar}^2, \quad V_H(r) = \sum_i \int U_\alpha(\vec{r}_i)V(\vec{r}_{ij})U(\vec{r}_i)d\vec{r}_i.$$

We seek to solve to solve this equation including mean interaction potential $V_H(r)$ between particles. We can use the central model potential $V_0(r)$ and corresponding analytical physical $U_{nL}$ and nonphysical $F_{nL}$ solutions. The negative eigenvalues of energies $E_\alpha = E_{nL} + \Delta E_{nL}$ are expressed by eigenvalues $E_{nL}$ for model potential. Substituting here, $U_\alpha(r)$ we have eigenfunctions and eigenvalues

$$U_\alpha = U_{nL} + \frac{U_{nL}}{W_0}\int_0^r F_{nL}CV_\delta U_\alpha dr - \frac{F_{nL}}{W_0}\int_0^r U_{nL}CV_\delta U_\alpha dr,$$

$$V_\delta(r) = V_H(r) - \Delta E_{nL}, \quad \Delta E_{nL} = \frac{1}{D}\int_0^\infty U_{nL}V_\delta U_\alpha dr, \quad D = \int_0^\infty U_{nL}U_\alpha dr$$

where boundary conditions $\lim_{r\to 0}\varphi_\alpha = 1$ and $\lim_{r\to\infty}\varphi_\alpha U_{nL} = 0$ are included. The eigenfunctions $U_\alpha(r)$ and eigenvalues $E_\alpha$ can be obtained by solving these equations simultaneously using the iteration method. For the first iteration, we must take $\varphi_\alpha = 1$ and $\Delta E_\alpha = 0$.

**REFERENCES**

[1] A. J. Janavičius, R. Bagdonaitė. Semi-relativistic nuclear shell model. *Lithuanian Journal of Physics*, **41** (3):150–153, 2001.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# THE GENERAL SOLUTION OF THE SCHRÖDINGER EQUATION FOR BOUND STATES

A. J. JANAVIČIUS and K. ŽILINSKAS

*Šiauliai University*

P. Višinskio 19, LT-77156, Šiauliai, Lithuania

E-mail: AYanavy@gmail.com, kest.zil@gmail.com

A general solution of the Schrödinger equation in the potential representation is put forward in the form of integral equations. The radial eigenfunctions in the potential representation of Schrödinger equation [1] can be expressed $U_\alpha(r) = \varphi_\alpha(r)U_{nL}(r)$ as a product of the unperturbed wave function $U_{nL}$ and multiplier $\varphi_\alpha$, which depends on the perturbation potential. In the Schrödinger equation

$$\frac{d^2}{dr^2}U_\alpha - \frac{L(L+1)}{r^2}U_\alpha + C\Big(E_\alpha - V_0(r) - V_\delta(r)\Big)U_\alpha = 0, \quad V_\delta(r) = V(r) - V_0, \quad C = \frac{2m}{\hbar^2}$$

which we seek to solve for potential $V(r)$, we can use the model potential $V_0(r)$ and corresponding analytical solutions $U_{nL}$. The negative eigenvalues of energies $E_\alpha = E_{nL} + \Delta E_{nL}$ can be expressed by eigenvalues $E_{nL}$ for model potential. Substituting here, $U_\alpha$ we have

$$\frac{d}{dr}(U_{nL}^2\frac{d}{dr}\varphi_\alpha) = -CU_{nL}\Big(\Delta E_{nL} - V_\delta(r)\Big)\varphi_\alpha U_{nL},$$

$$\varphi_\alpha U_{nL} = U_{nL} - C\int_0^r \frac{dy}{U_{nL}^2}\int_0^y U_{nL}\Big(\Delta E_{nL} - V_\delta(r)\Big)\varphi_\alpha U_{nL}dz, \quad \Delta E_{nL} = \frac{\int_0^\infty U_{nL}V_\delta(r)\varphi_\alpha U_{nL}dr}{\int_0^\infty U_{nL}\varphi_\alpha U_{nL}dr}$$

where boundary conditions $\lim_{r\to 0}\varphi_\alpha = 1$ and $\lim_{r\to\infty}\varphi_\alpha U_{nL} = 0$ are included. The eigenfunctions $U_\alpha(r)$ and eigenvalue $E_\alpha$ can be obtained by solving these equations simultaneously using the iteration method. For the first iteration, we must take $\varphi_\alpha = 1$ and $\Delta E_\alpha = 0$.

**REFERENCES**

[1] A. J. Janavičius. The general solution of the Schrödinger equation for positive energies and bound states in the one-particle case. *Lithuanian Journal of Physics*, **38** (5):437–441, 1998.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# NUMERICAL ANALYSIS OF THE THERMALIZATION PROCESS IN GOLD FILMS IRRADIATED BY FEMTOSECOND LASER PULSES BY USING PARABOLIC AND HYPERBOLIC TWO TEMPERATURE MODELS

GERDA JANKEVIČIŪTĖ[1], RAIMONDAS ČIEGIS[1], ALEKSANDR DEMENTJEV[2] and KĘSTUTIS RAČKAITIS[2]

[1] *Vilnius Gediminas Technical University*

Saulėtekio av. 11, LT-10223 Vilnius, Lithuania

E-mail: rc@vgtu.lt

[2] *Center for Physical Sciences and Technology (Institute of Physics)*

Savanorių 231, LT-02300, Vilnius, Lithuania

E-mail: aldement@ktl.mii.lt

Ultrashort lasers have opened a very promising research area in the micro- and nanoscale regimes due to their ability to deposit extremely high density energy into a target in a very short time. It was understood that standard Fourier's law of heat conduction is not well suited for modeling of ultrafast heat transport processes, in which the energy deposition times are shorter than or comparable to the relaxation times of heat carriers [1]. For that reason, the two temperature model (TTM) have been proposed (1974) by Anisimov *et al* for describing the process of ultrashort laser pulses interaction with metals [1; 2]. The electron and lattice subsystems were represented by an electron and lattice temperatures. It is assumed in this parabolic two-step (PTS) model that thermalizations of the electron and phonon subsystems occur instantaneously. For the accounting of electron and phonon subsystems relaxation times the hyperbolic two-step (HTS) model was introduced later [1]. In both models the thermalization between these subsystems is realized through the coupling term $G(T_e - T_l)$. Therefore, it was implicitly assumed for a long time that the electron temperature is anywhere higher than lattice temperature during this relaxation process [2]. We show in this paper by using both PTS and HTS models [3] that due to the fast transport of energy by the electron heat conduction from the subsurface layer into the deeper layers of metals the temperature of electrons in the enough thick subsurface layer can become lower than the lattice temperature if even the heat conduction in the lattice is taken into account.

**REFERENCES**

[1] J. Dowden. The Theory of Laser Materials processing: Heat and Mass Transfer in Modern Technology, Springer 2009; C.P. Grigoropoulos, Transport in Laser Microfabrication: Fundamentals and Applications. CUP, 2009; D.Maria, U.V.Bhandarkar, S.S. Joshi. Critical assessment of the issues in the modeling of ablation and plasma expansion processes in the pulsed laser deposition of metals. J. Appl. Phys, 109: 021101, 2011.

[2] G. Du, F. Chen, Q. Yang, J. Si, X. Hou. Ultrafast temperature relaxation evolution in Au film under femtosecond laser pulses irradiation. Opt. Commun. 283(9):1869-1872, 2010; Ultrafast thermalization characteristics in Au film irradiated by temporally shaped femtosecond laser pulses. Opt. Commun. 284(2): 640-645, 2011.

[3] R. Čiegis, A. Dement'ev, G. Jankevičiūtė. Numerical analysis of hyperbolic two-temperature model. Lithuanian Math. J. 48(1): 46-60, 2008.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# COMPUTING THE ASYMPTOTIC EXPANSION OF THE MEDIAN OF THE ERLANG DISTRIBUTION

P. JODRÁ

*Universidad de Zaragoza*

María de Luna 3, 50018 Zaragoza, Spain

E-mail: pjodra@unizar.es

In this work, we develop an algorithm for computing the asymptotic expansion of the median of the Erlang distribution. The median $\lambda_n$ of the Erlang distribution $X_n$ with shape parameter $n+1$ and scale parameter 1 is defined as the only solution of the equation $F_n(\lambda_n) = 1/2$, where $F_n(x) := P(X_n \leq x) = (1/\Gamma(n+1)) \int_0^x t^n e^{-t} dt$ for $x \geq 0$ and $n = 0, 1, 2, \ldots$. Analytical solutions of this equation are known only for the cases $n = 0$ ($\lambda_0 = \log 2$) and $n = 1$ ($\lambda_1 = -W_{-1}(-1/2e) - 1$, cf. Jiménez and Jodrá [3] for details).

Choi [2] presents a procedure for computing the asymptotic expansion of the median of $X_n$, that is, for computing the rational coefficients $q_i$ in the expansion $\lambda_n = n + 2/3 + \sum_{i=1}^{r} q_i/n^i + O\left(1/n^{r+1}\right)$. Choi's procedure is based on a connection between $\lambda_n$ and a sequence $(\theta_n)_{n\geq 0}$, which was introduced by Ramanujan [4], through the equality: $1 - \theta_n = (e^n/n^n) \int_n^{\lambda_n} t^n e^{-t} dt$, $n = 1, 2, \ldots$. We are interested in computing a large number of coefficients $q_i$ in the above asymptotic expansion because these terms can be used to obtain rational bounds of $\lambda_n$ as it was shown in Adell and Jodrá [1], where were given the first seven coefficients.

Our algorithm is an improved version of Choi's procedure and we have implemented it in Maple Release 13. In view of the numerical results obtained, we can answer the following questions: (i) Does the series $\{|q_i|\}_{i\geq 1}$ converge to 0 such as numerical results in [1] may be suggesting? (ii) Which is the partial sum $n + 2/3 + \sum_{i=1}^{r} q_i/n^i$ closest to the true value of $\lambda_n$ as well as its number of significant digits?

*Acknowledgments.* This research has been funded by DGA (Grupo PDIE) and by Vicerrectorado de Investigación (Universidad de Zaragoza) project 226/128.

**REFERENCES**

[1] J.A. Adell and P. Jodrá. On a Ramanujan equation connected with the median of the gamma distribution. *Trans. Am. Math. Soc.*, **360** (7):3631–3644, 2008.

[2] K.P. Choi. On the medians of gamma distributions and an equation of Ramanujan. *Proc. Amer. Math. Soc.*, **121** (1):245–251, 1994.

[3] F. Jiménez and P. Jodrá. On the computer generation of the Erlang and negative binomial distributions with shape parameter equal to two. *Math. Comput. Simul.*, **79** (5):1636–1640, 2009.

[4] S. Ramanujan. Question 294. *J. Indian Math. Soc.*, **3** :128, 1911.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON THE FUNCTIONAL INDEPENDENCE OF DIRICHLET $L$-FUNCTION AND PERIODIC HURWITZ ZETA-FUNCTION

ROMA KAČINSKAITĖ

*Šiauliai University*

P. Višinskio str. 19, Šiauliai LT-77156, Lithuania

E-mail: r.kacinskaite@fm.su.lt

The problem of the independence of functions come back to O. Hölder and D. Hilbert. In [1], O. Hölder proved the algebraic-differential independence for the gamma-function $\Gamma(s)$. In [2], S. M. Voronin obtained the functional independence of the Riemann zeta-function $\zeta(s)$.

In this talk, we present the functional independence for the Dirichlet $L$-function and the periodic Hurwitz zeta-function.

Let $\alpha$, $0 < \alpha \leq 1$, be a fixed number, and $\mathfrak{b} = \{b(m) : m \in \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers $b(m)$ with a minimal period $l \in \mathbb{N}$. For $\sigma > 1$, the periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{b})$ and the Dirichlet $L$-function $L(s, \chi)$ are defined by the series

$$\zeta(s, \alpha; \mathfrak{b}) = \sum_{m=0}^{\infty} \frac{b(m)}{(m+\alpha)^s} \quad \text{and} \quad L(s, \chi) = \sum_{m=1}^{\infty} \frac{\chi(m)}{m^s},$$

respectively.

THEOREM 1. *Suppose that $\alpha$ is transcendental number. Let, for every $j = 1, \dots, n$, $F_j : \mathbb{C}^{2N} \to \mathbb{C}$ be a continuous function, $N \in \mathbb{N}$, and*

$$\sum_{j=0}^{n} s^j F_j(L(s, \chi), L'(s, \chi), \dots, L^{(N-1)}(s, \chi), \zeta(s, \alpha; \mathfrak{b}), \zeta''(s, \alpha; \mathfrak{b}), \dots, \zeta^{(N-1)}(s, \alpha; \mathfrak{b}) \equiv 0.$$

*Then $F_j \equiv 0$ for $j = 1, \dots, n$.*

**REFERENCES**

[1] O. Hölder. Über die Eigenschaft der Gammafunktion keiner algebraischen Differentialgleichung zu genügen. *Math. Ann.*, **28** : 1–13, 1887.

[2] S. M. Voronin. On differential independence of $\zeta(s)$ functions. *Dokl. Akad. Nauk SSSR*, **209** (6): 1264–1266, 1973. (in Russian) = *Sov. Math. Dokl.,* **14** : 607–609, 1973.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# INTEGRAL-LIKE $T$- AND $S$-EVALUATORS[1]

MARTIN KALINA

*Dept. of Mathematics, Slovak University of Technology*

Radlinského 11, Sk-813 68n Bratislava, Slovakia

E-mail: kalina@math.sk

$T_L$- and $S_L$-evaluators were defined in[1] as extentions of $T_L$-filters and their duals to bounded lattices. Later on, in [2] general $T$- and $S$-evaluators (where $T$ stands for a t-norm $T$ and $S$ for a t-conorm $S$, see[5]) were proposed. $T$-evaluators are a natural tool for defining $T$-transitive fuzzy-relations, i.e., relations $R : X^2 \to [0, 1]$ fulfilling the following

$$T(R(x, y), R(y, z)) \leq R(x, z).$$

This $T$-transitivity might be important, e.g., in preference modelling (see [4]). In [3] a connection between different kinds of integrals on one hand, and $T$- and $S$-evaluators on the other hand, were studied. In this contribution we want to investigate how far it is possible to go in generalizing the notion of integral to preserve the properties of $T$- and/or $S$-evaluators.

**REFERENCES**

[1] S. Bodjanova. $T_L$- and $S_L$-evaluators. In: *Proceeding of AGOP 2007*, Academia Press, Ghent,, 2007, 165–172.

[2] S. Bodjanova, M. Kalina. $T$-evaluators and $S$-evaluators. *Fuzzy Sets and Systems*, **160** (2009), 1965–1983.

[3] S. Bodjanova, M. Kalina. Sugeno and Shilkret integrals, and $T$- and $S$-evaluators. In: *Proceeding of 7-th International Symposium on Inteligent Systems and Informatics, SISY 2009*, Subotica, 2009, 109–114.

[4] D. Hliněná, M. Kalina, P. Kráľ. Choquet integral with respect to Łukasiewicz filters, and its modifications. *Information Sciences*, **179** (2009), 2912–2922.

[5] E.P. Klement, R. Mesiar, E. Pap . Triangular norms *Kluwer*. Dordrecht, 2000.

[1]The support of Science and Technology Assistance Agency under the contract No. APVV-0073-10, and of the VEGA grant agency, grant numbers 1/0143/11 and 1/0080/10, is kindly acknowledged.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ABOUT MATHEMATICAL MODELLING OF PEAT BLOCKS IN 3-LAYERED 3D DOMAIN[1]

ILMĀRS KANGRO[1], AIGARS GEDROICS[2] and HARIJS KALIS[2,3]

[1] *Rēzekne Higher Education Institution, Departament of engineering science*
Atbrīvošanas aleja 90, Rēzekne, LV-4601, Latvia

[2] *Department of Mathematics, University of Latvia*
Zeļļu iela 8, Rīga, LV-1002, Latvia

[3] *Institute of Mathematics and Computer Science of University of Latvia*
Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: kangro@cs.ru.lv, kalis@lanet.lv

In this paper we consider averaging with integral parabolic splines and finite difference methods for solving the 3-D boundary-value problem in multilayered domain. The mathematical model for calculation of concentration of metal in the 3-layered peat block is developed. A specific feature of these problems is that it is necessary to solve the 3-D boundary-value problems for elliptic type partial differential equations (PDEs) of second order with piece-wise diffusion coefficients in three layer domain. We develop here a finite-difference method for solving of a problem of the above type with periodical boundary condition in x direction. This procedure allows to reduce the 3-D problem of PDEs to a system of 2-D problems of PDSs by using circulant matrix. The operations with circulant block- matrices and vectors of the third order are obtained. The numerical results with experimental data are compared.

[1]This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund and by the grant 09.1572 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON DAMPED GAUSS-NEWTON-TYPE METHODS FOR PARAMETER IDENTIFICATION

INGA KANGRO and OTU VAARMANN

*Institute of Cybernetics at Tallinn University of Technology*

Akadeemia tee 21, 12618 Tallinn, Estonia

E-mail: inga@ioc.ee, vaarmann@ioc.ee

Usually we do not have a ready-made mathematical model waiting for solution rather, it may include some parameters which values are not known. Thus an important problem in modelling is that of parameter identification from available performance data. Inverse problems are, as usual, ill-posed because they typically involve the estimation of certain quantities based on indirect measurments. Besides modelling based on economical, technical and environmental criteria include many sources of uncertainty and subjectivity. This matter has stimulated a study of new, robust and heuristics concepts of approximation and optimization. A trained feedforward artificial neural network is capable of approximating an unknown mapping. Different structures of artificial neural network based identification are considered as approximation tools for identification of complex nonlinear systems and processes, e.g., involving higly nonlinear, high dimensional, noisy functions.

In most application the activation (transfer) function is a logistic or hyperbolic tangent function. The main advantage of using sigmoid functions is that they are always differentiable and it is very easy and fast to calculate the derivatives of these functions. That is why we consider here multilayer feedforward neural networks with the Levenberg-Marquardt-type learning rule which is much faster and reliable than the traditional steepest descent rule. The training is based on a set of input-output data and the aim of the training is to minimize a cost (evaluation) function. Usually mean squares error is used as the minimization criterion for adjusting the weights to minimize the classification error of the network of neurons that leads to solving of highly nonlinear least squares problems. For solving of those, regularizing algorithms containing several adjusting parameters may be fruitful, because a proper choice of additional parameters enables to improve the convergence properties and to reduce the amount of computation.

Here two-parameter iterative regularization methods based on the Gauss-Newton method under certain condition on a test function and the required solution are studied. To get more realistic impression of convergence properties of the methods under discussion their approximate variants are considered, their computational aspects are discussed and a local convergence theorem is proposed [1].

**REFERENCES**

[1] I. Kangro and O. Vaarmann . Some iterative regularized methods for highly nonlinear least squares problems . *Mathematical Modelling and Analysis* , **14** (2):179–186, 2009 .

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# OPTIMAL SUPERCONVERGENCE RATE FOR SOLVING WEAKLY SINGULAR FREDHOLM INTEGRAL EQUATIONS

RAUL KANGRO[1] and INGA KANGRO[2]

[1] *Institute of Mathematical Statistics, University of Tartu*

Liivi 2-217, 50409, Tartu, Estonia

E-mail: raul.kangro@ut.ee

[2] *Institute of Cybernetics, Tallinn University of Technology*

Akadeemia 21, 12618, Tallinn, Estonia

E-mail: inga@ioc.ee

Piecewise polynomial collocation method is one of the most popular methods for solving Fredholm integral equations

$$y(t) = \int_0^b K(t,s)y(s)ds + f(t), \qquad t \in [0,b].$$

It is known that often in the case of such methods the maximal error of the approximate solution at the collocation points convergences to zero more quickly than the maximal error over the whole interval, provided that some additional restrictions on the choise of the collocation parameters are imposed. In the case of weakly singular equations one can improve both the global convergence rate and the superconvergence rate at the collocation points by using a graded grid of the form

$$s_n = \frac{b}{2}\left(\frac{n}{N}\right)^r, \quad n = 0,1,\ldots,N,$$

$$s_{N+n} = b - s_{N-n}, \quad n = 1,\ldots,N$$

with a suitable nonuniformity parameter $r \geq 1$ (see e.g. [1], [2]). The best known superconvergence results state that the maximal convergence order is achieved for $r \geq r_0$ where $r_0$ is a certain number depending on the smoothness of the kernel and data. In the talk we present results which give optimal bounds for the superconvergence rate for all values of the nonuniformity parameter $r$. It follows from these bounds that the maximal convergrnce order is achieved for smaller values of $r$ than stated in the previously known results.

**REFERENCES**

[1] A. Pedas, G. Vainikko. Superconvergence of piecewise polynomial collocations for nonlinear weakly singular integral equations. *J. Integral Equations Appl.*, **9** (4). 1997, 379 - 406

[2] G.Vainikko. *Multidimensional Weakly Singular Integral Equations*. Lecture Notes Math., 1549, Springer-Verlag, Berlin-Heidelberg-New York, 1993.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# CONVERGENCE OF COLLOCATION METHOD WITH DELTA FUNCTIONS FOR INTEGRAL EQUATIONS OF FIRST KIND

URVE KANGRO

*Department of Mathematics and Computer Science*
*University of Tartu*

J. Liivi 2, Tartu 50409, ESTONIA

E-mail: urve.kangro@ut.ee

We consider integral equations of type

$$\int_\gamma K(x,y)u(y)dS_y = f(x), \quad x \in \Gamma, \tag{1}$$

where $\gamma$ and $\Gamma$ are some closed disjoint curves or surfaces. Equations of this type arise when solving boundary value problems of elliptic partial differential equations by interior source methods. These methods generate the solution of the differential equation as an integral over a contour or a surface outside the closure of the (usually exterior) domain. Typically $K$ has a singularity at $x = y$, and if $\Gamma$ and $\gamma$ are disjoint, the singularity is avoided. In fact, if $\Gamma$ and $\gamma$ are analytic, then the integral equation (1) has an analytic kernel. Results about existence and uniqueness of the solution can often be obtained only in spaces of linear analytic functionals, and in general case, only density of the range of the integral operator can be proved.

We look for approximate solutions of the integral equation as linear combinations of Dirac's $\delta$-functions. This particular form is motivated by simplicity of the corresponding solution of the differential equation. Only the solution of the differential equation is of interest ($u$ has no physical meaning), hence one only needs the residual to be small. For the corresponding collocation method, in case of analytic data the convergence is very fast (exponential in the number of variables), and the method converges even for boundaries with corners, if one chooses the interior boundary, the supports of the $\delta$-functions and the collocation points carefully. On the other hand, the collocation method is sensitive to the choice of the interior boundary, and the location of the points. An alternative approach is to choose on $\Gamma$ more points than on $\gamma$, and solve the corresponding overdetermined system by least squares.

**REFERENCES**

[1] U. Kangro. Convergence of Collocation Method with Delta Functions for Integral Equations of First Kind. *Integr. equ. oper. theory*, **66** (2):265–282, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# PROPAGATION OF ELECTROMAGNETIC SOLITON GOVERNED BY THE COUPLED MAXWELL AND THE LANDAU-LIFSHITZ EQUATIONS

LOUIS KAVITHA[1*], MANICKAM SARAVANAN[1], BALASUBRAMANIAM SRIVIDYA[1] and DHANARAJ GOPI[2]

[1] *Department of Physics, Periyar University*
[2] *Department of Chemistry, Periyar University*

Salem 636 011, Tamilnadu, India

E-mail: kavithalouis@yahoo.com

The antisymmetric exchange interaction also called Dzyaloshinskii-Moriya (DM), allowed in particular lattice site is of interest in its own right. In order to investigate the effect of Dzyaloshinskii-Moriya and the single ion anisotropy in a classical Heisenberg antiferromagnet under the influence of electromagnetic field, the associated Landau-Lifshitz Maxwell equation

$$\frac{\partial \mathbf{M}}{\partial t} = \mathbf{M} \times \Big[\gamma \mathbf{H} - J\lambda \frac{\partial \mathbf{M}}{\partial z} - AM^z \hat{n} - \lambda D^z \frac{\partial (M^y \hat{i} - M^x \hat{j})}{\partial z}\Big]. \tag{1}$$

$$c^2 \frac{\partial^2 \mathbf{B}}{\partial z^2} - \frac{\partial^2 \mathbf{B}}{\partial t^2} = \frac{1}{\epsilon_0}\Big[\frac{\partial^2 \mathbf{M}}{\partial z^2} - \frac{\partial^2 M^z}{\partial z^2}\hat{\mathbf{n}}\Big]. \tag{2}$$

is solved through reductive perturbation technique. It is found that the dynamics of the modeled system is governed by derivative nonlinear Schrödinger equations when the parameter of the DM interaction is altered by proper tuning, along the propagation of the electromagnetic waves. The investigation of the exact solution, in particular solitons, for the nonlinear evolution equations plays an important and interesting role in the study of nonlinear physical phenomenon. Thus an effective algebraic method which is straightforward and systematic to find the exact solution is employed to identify the soliton solution pattern of the derivative nonlinear Schrödinger equation using symbolic computation. Interesting finite energy solutions including multi-solitons have been obtained in these systems only in certain limiting cases. This type of physically interesting nonlinear evolutions is found to have potential applications in the areas of magneto-optical recording devices for higher density data storage and switching mechanisms.

**REFERENCES**

[1] M. Daniel and L. Kavitha. Localized spin excitation in an anisotropic Heisenberg ferromagnet with Dzyaloshinskii-Moriya interactions. *Phys. Rev. B*, **63** :172302 (1-4), 2001.

[2] M. Daniel and L. Kavitha. Soliton in an anisotropy spinladder with site and spin-dependent Dzyaloshinsky- Moriya interaction. *Phys. Letts. A*, **295** :121-132, 2002.

[3] L. Kavitha, P. Sathishkumar, M. Saravanan and D. Gopi. Magnetization reversal through soliton in a site-dependent weak ferromagnet. *Journal of Physical Society, Japan*, (Accepted for publication (2011)).

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON DEPARTURE COUNTING PROCESS IN THE BATCH ARRIVAL QUEUEING SYSTEM WITH $N$-POLICY AND MULTIPLE VACATIONS

WOJCIECH M. KEMPA

*Silesian University of Technology, Institute of Mathematics*

ul. Kaszubska 23, 44-100 Gliwice, Poland

E-mail: wojciech.kempa@polsl.pl

Queueing systems with server vacations are good models for different real-life situations occurring e.g. in manufacturing, telecommunication and computer networks (see e.g. [4]). In the paper we study an infinite-buffer queueing system with the classical multiple vacation policy extended by the $N$-policy. It is assumed that customers arrive in batches of random sizes according to a Poisson process and are served individually with FIFO service discipline. After each busy period the server begins a vacation period consisting of a number of independent single vacations. Successive single vacations are initiated as far as, at the end of one of them, the number of customers in the system equals at least $N$. Then, that single vacation is the last one and at its completion epoch the next busy period begins.

One of the main characteristics helpful in performance evaluation of each queueing system is departure counting process $h(t)$ that at any fixed time $t$ takes on a random value equal to the number of customers completely served before $t$. This characteristic was studied in [1] for the general-type queueing system without vacations, and in [2] and [3] for other types of vacation policies.

In the paper we investigate the transient distribution of departure counting process using the approach consisting of two main stages. Firstly we deal with an auxiliary system that starts working being empty and waits for customers. We come down the analysis of $h(t)$ in such a system to the case of the "pure" system (without vacation policies) using the formula of total probability. Next, introducing a delayed renewal process of successive vacation cycles, we go to the original system.

As a main result the explicit representation for the expression

$$\sum_{m=0}^{\infty} z^m \int_0^{\infty} e^{-st}\mathbf{P}\{h(t)=m\}\,dt, \qquad |z|<1,\ \ \mathrm{Re}(s)>0 \tag{1}$$

is obtained in terms of transforms of "input" distributions of the system and components of Wiener-Hopf factorization identity connected with them.

**REFERENCES**

[1] W.M. Kempa. The departure process for queueing systems with batch arrival of customers. *Stoch. Models*, **24** (2):246–263, 2008.

[2] W.M. Kempa. On one approach to study of departure process in batch arrival queues with multiple vacations. In: *Proc. of the ASMDA*, 2009, 203–207, .

[3] W.M. Kempa. Some new results for departure process in the $M^X/G/1$ queueing system with a single vacation and exhaustive service. *Stoch. Anal. Appl.*, **28** (1):26–43, 2010.

[4] H. Takagi. *Queueing analysis: a foundation of performance evaluation. Vol. 1: Vacation and priority systems.* North-Holland, Amsterdam - New York - Oxford - Tokyo, 1991.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# RARE PERIODIC AND CHAOTIC ATTRACTORS IN THREE-TIMES ITERATED IKEDA MAP

ALEX V. KLOKOV and MIKHAIL V. ZAKRZHEVSKY

*Riga Technical University, Institute of Mechanics*

Kaļķu iela 1, Rīga, LV-1658, Latvia

E-mail: alex_klokov@inbox.lv

**Keywords:** *nonlinear discrete model, complete bifurcation analysis, method of complete bifurcation groups, rare attractors, unstable periodic infinitium, chaos, domains of attraction.*

A new approach for the global bifurcation analysis, based on the ideas of Poincare, Birkhoff and Andronov, of nonlinear systems, described by discrete equations, is under consideration. The main idea of the approach is a concept of complete bifurcation groups and periodic branch continuation along stable and unstable solutions, named by one of the authors as a method of complete bifurcation groups (MCBG) [1; 2; 3; 4]. Rare attractors in difference nonlinear models, e.g. [5], can be found using the same approaches as for nonlinear ODE [4]. As examples we discuss using the method of complete bifurcation group for three-times iterated Ikeda map [5]:

$$\begin{cases} x_{n+1} = a + b(x_n cos(\omega_n) - y_n sin(\omega_n)), \\ y_{n+1} = b(x_n sin(\omega_n) + y_n cos(\omega_n)), \end{cases} \tag{1}$$

where

$$\omega_n = c - \frac{d}{1 + x_n^2 + y_n^2}. \tag{2}$$

The main results are presented by complete bifurcation diagrams for variable parameter b of the system. We have found different new rare regular and chaotic attractors and some other new nonlinear phenomena such as cluster of submerged subharmonic isles, fully unstable subharmonic isle and multiplicity of chaotic attractor and usual of period-1 attractor. It is shown, that the parameter b of the three-times iterated Ikeda map is sufficient for global topology of the steady-state solutions of the system. All results were obtained numerically, using software SPRING [2], created in Riga Technical University.

## REFERENCES

[1] M. Zakrzhevsky. New concepts of nonlinear dynamics: complete bifurcation groups, protuberances, unstable periodic infinitiums and rare attractors. *Journal of Vibroengineering - JVE 10*, **10** (4):421–441, 2008.

[2] I. Schukin. *Development of the methods and algorithms of simulation of nonlinear dynamics problems. Bifurcations, chaos and rare attractors.* PhD Thesis, Riga - Daugavpils, Latvia, 205 p., 2005.

[3] M. Zakrzhevsky. Global Nonlinear Dynamics Based on the Method of Complete Bifurcation Groups and Rare Attractors. In: *Proceedings of the ASME 2009 (IDETC/CIE 2009)*, San Diego, USA, 2009, CD, 8 p.

[4] M. Zakrzhevsky. How to Find Rare Periodic and Rare Chaotic Attractors?. In: *Abstracts of the 8th AIMS International Conference, http://www.math.tudresden.de/koksch/AIMS/*, Dresden, Germany, 2010, 9 p.

[5] H. Ito, Y. Mochida, A. Kumamoto. Detecting Unstable Periodic Orbits in Hyperchaotic Systems Using Subspace Fixed-Point Iteration. *Journal of Measurement Science and Instrumentation*, **1** (Suppplement): 53–56, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ANALYTICAL SOLUTIONS OF EDDY CURRENT PROBLEMS IN CYLINDRICAL COORDINATES

VALENTINA KOLISKINA and INTA VOLODKO

*Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

E-mail: v.koliskina@gmail.com, inta.volodko@rtu.lv

Analytical solutions of eddy current problems for planar multilayer media are well-known in the literature for constant properties of a conducting medium (see, for example, [1]). Similar problems for coils encircling multiple coaxial conductors or coils inside multiple coaxial conductors are solved in [2]. In some industrial processes the properties of a conducting medium (electric conductivity and magnetic permeability) are modified and may depend on geometrical coordinates. As a result, there is a need to develop mathematical models of eddy current testing problems with varying properties of the medium.

In the present paper we construct analytical solutions of eddy current problems in cylindrical coordinates where a coil with alternating current is located inside or outside a a multilayer tube. It is assumed that the electric conductivity and magnetic permeability of conducting layers of the tube are power functions of the radial coordinate. The problem is solved by the method of the Fourier transform. The system of ordinary differential equations for the vector potential in the transformed space can be solved in terms of different special functions. Two examples are considered in detail: (1) a coil outside a two-layer tube; (2) a coil inside a two-layer tube. It is assumed in both cases that the properties of one layer vary with respect to the radial coordinate while the properties of the other layer are assumed to be constant. Results of numerical calculations are presented.

**REFERENCES**

[1] J.A. Tegopoulos and E.E. Kriezis. *Eddy currents in linear conducting media.* Elsevier, New York, 1985.

[2] C.V. Dodd, C.C. Cheng, and W.E. Deeds. Induction coils coaxial with an arbitrary number of cylindrical conductors. *J. Appl. Phys.*, **45:** 638–647, 1974.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# VALUE-DISTRIBUTION OF TWISTED $L$-FUNCTIONS OF NORMALIZED HECKE EIGEN FORMS

ALESIA KOLUPAYEVA

*Šiauliai University*

P. Višinskio 19, LT 77156 Šiauliai , Lithuania

*Vilnius University*

Naugarduko 24, LT 03225 Vilnius, Lithuania

E-mail: alesia.su@gmail.com

Suppose that $F(z)$ is a holomorphic normalized Hecke eigen cusp form of weight $\kappa$ for the full modular group. Let

$$F(z) = \sum_{m=1}^{\infty} c(m)\, e^{2\pi i m z}, \quad c(1) = 1,$$

be the Fourier series expansion at $\infty$ for $F(z)$.

Let $\chi(m)$ denote a Dirichlet character modulo $q \in \mathbb{N}$. Then the twisted $L$-function $L(s, F, \chi)$, $s = \sigma + it$, attached to the form $F(z)$ is defined, for $\sigma > \frac{\kappa+1}{2}$, by the Dirichlet serias

$$L(s, F, \chi) = \sum_{m=1}^{\infty} \frac{c(m)\chi(m)}{m^s},$$

and can be analytically continued to an entire function.

We consider the value distribution of $L(s, F, \chi)$ when the module $q$ is prime and is increasing. The first results in the field were obtained in [1], [2]. For $Q \geq 2$, define $M_Q = \sum\limits_{q \leq Q} \sum\limits_{\substack{\chi = \chi (\mathrm{mod} q) \\ \chi \neq \chi_0}} 1$, where $\chi_0$ is the principal character.

THEOREM 1. *Suppose that $\sigma > \frac{\kappa+1}{2}$. Then the probability measure*

$$M_Q^{-1} \sum_{q \leq Q} \sum_{\substack{\chi = \chi (\mathrm{mod} q) \\ \chi \neq \chi_0 \\ L(s,F,\chi) \in A}} 1, \quad A \in \mathcal{B}(\mathbb{C}),$$

*converges weakly to the probability measure defined by characteristic transform as $Q \to \infty$.*

## REFERENCES

[1] A. Kolupayeva and A. Laurinčikas. Value-distribution of twisted automorphic $L$-functions. *Lith. Math. J.*, **48** (2):203–211, 2008.

[2] A. Kolupayeva and A. Laurinčikas. Value-distribution of twisted automorphic $L$-functions. II . *Lith. Math. J.*, **50** (3):284–292, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# COMPARISON OF THE SOME ALGEBRAIC DOMAIN DECOMPOSITION BASED PRECONDITIONERS ON MANYCORES

PAWAN KUMAR

*Department of Applied sciences*

Université libre de Bruxelles, Bruxelles

E-mail: kumar.lri@gmail.com

The problem of solving large sparse linear system of the form

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{1}$$

by preconditioned iterative method is considered. Here $\mathbf{A} \in \mathbb{R}^{N\times N}$, $\mathbf{x} \in \mathbb{R}^N$, $\mathbf{b} \in \mathbb{R}^N$. Using a popular graph partitioner like the one described in [2], the adjacency graph of the matrix $\mathbf{A}$ is partitioned and reordered. Three preconditioners namely NSSOR, NMILUR, and NMILUC as described in [3] are multithreaded using Cilk++, and the effectiveness of these methods are tested on many cores for a range of problems coming from convection-diffusion and Florida matrix market collection [1].

**REFERENCES**

[1] T. A. Davis and Y. Hu. The university of Florida sparse sparse matrix collection. *ACM Transaction on mathematical software*, to appear.

[2] G. Karypis and V. Kumar. A fast and high quality multilevel scheme for partitioning irregular graphs. *SIAM J. Sci. Comp.*, **1** (2):359-392, 1999.

[3] L. Grigori, P. Kumar, F. Nataf and K. Wang. A class of multilevel parallel preconditioning strategies. *http://hal.archives-ouvertes.fr/inria-00524110/fr/*, .

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON SOME DIFFERENTIAL EQUATIONS FROM APPLICATIONS

DMITRIY LAPIDUS

*Daugavpils University*

Parādes iela 1, Daugavpils, LV-5400, Latvia

E-mail: lapidus1985@inbox.lv

Differential equations occurring in applications are discussed and analysed. Some simplified examples are treated.

Kinetics of amyloid formations. Relative to the fibril concentration, the infinite set of reactions (R1) and (R2) is described by two coupled differential equations (1).

This approach has been used for example, to analyze kinetic data for polyglutamine peptide aggregation; In an alternative embodiment of the nucleated polymerization model, the kinetics of nucleation are postulated to be slow and essentially irreversible:

$$\begin{cases} -x' &= \alpha xy, \\ y' &= \gamma x^n. \end{cases} \tag{1}$$

Related to Medicine. Simulated kinetic profiles were generated using finite difference methods in fiber formation process (2).

$$\begin{cases} x' &= \alpha y(x - \alpha x), \\ y' &= -\alpha y(x + \beta x) - n\gamma y^n - \delta yz - \epsilon yu, \\ z' &= \gamma y^n + \epsilon yz - k_1 z(v + k_2 w), \\ v' &= -k_1 z(v + k_2 w), \\ w' &= k_1 z(v - k_2 w). \end{cases} \tag{2}$$

**REFERENCES**

[1] Regina M. Murphy. Kinetics of amyloid formation and membrane interaction with amyloidogenic proteins. *Biochimica et Biophysica Acta*, 1923–1934, 2007.

[2] Jennifer L. Larson and Andrew D. Miranker. The Mechanism of Insulin Action on Islet Amyloid Polypeptide Fiber Formation. *J. Mol. Biology*, **335** 221–231, 2004.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# THE FINITE PART OF DIVERGENT INTEGRALS WITH LOGARITHMIC FACTORS

KAIDO LÄTT

*Institute of Mathematics, University of Tartu*

J.Liivi 2, 50409 Tartu, Estonia

E-mail: klatt@ut.ee

Equations with divergent integrals are used in a wide variety of applications in physics [1] and mathematics [2]. To study divergent integrals and equations containing them, the concept of the finite part (f.p.) of divergent integrals (the notion of f.p.-integrals) is established (see for example [3]). In [4], the formula for change of variables for f.p.-integrals based on divergent integrals of the form

$$\int_0^R a(r)r^{-\lambda-1}dr,$$

where $\lambda \in \mathbb{C}$ and $a$ is absolutely integrable, is proved.

We discuss the concept of change of variables in f.p.-integrals of divergent integrals of the form

$$\int_0^R a(r)r^{-\lambda-1}(\ln r)^n dr,$$

where $\lambda \in \mathbb{C}$, $n \in \mathbb{N}_0 = \{0, 1, 2, \dots\}$ and $a$ is absolutely integrable.

**REFERENCES**

[1] I. Lifanov, L. Poltavskii and G. Vainikko. *Hypersingular integral equations and their applications.* Chapman & Hall/CRC, Boca Raton, FL, 2004.

[2] J. Saranen and G. Vainikko. *Periodic integral and pseudodifferential equations with numerical approximation.* Springer-Verlag, Berlin, 2002.

[3] G. Vainikko and I. Lifanov. On the Notion of the Finite Part of Divergent Integrals in Integral Equations. *Differ. Equ.*, **38** (9):1313–1326, 2002.

[4] G. Vainikko and I. Lifanov. Approaches to the summability of divergent multidimensional integrals. *Sb. Math.*, **194** (7-8):1137–1166, 2003.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# APPROXIMATION OF ANALYTIC FUNCTIONS BY SHIFTS OF ZETA-FUNCTIONS

ANTANAS LAURINČIKAS

*Vilnius University*

Naugarduko 24, LT-03225 Vilnius, Lithuania

*Šiauliai University*

P. Višinskio 19, LT-77156 Šiauliai, Lithuania

E-mail: antanas.laurincikas@mif.vu.lt

Suppose that the function $f(s)$ is continuous on compact subset $K \subset \mathbb{C}$ and analytic in the interior of $K$. Then, by the Mergelyan theorem, for every $\varepsilon > 0$, there exists a polynomial such that

$$\sup_{s\in K} |f(s) - p(s)| < \varepsilon.$$

Since 1975 it is known that there exist analytic functions $g(s)$ such that their shifts $g(s + i\tau)$ approximate uniformly on compact subsets of some region with a given accuracy every analytic function. The first example of such functions is the Riemann zeta-function $\zeta(s)$. Suppose that $K$ is a compact subset of the strip $D = \{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$ with connected complement, and $f(s)$ is a continuous non-vanishing function on $K$ and analytic in the interior of $K$. Then, for every $\varepsilon > 0$,

$$\liminf_{T\to\infty} \frac{1}{T}\text{meas}\left\{\tau \in [0, T] : \sup_{s\in K} |\zeta(s + i\tau) - f(s)| < \varepsilon\right\} > 0.$$

In the report, we will discuss approximation of analytic functions by shifts $F(\zeta(s + i\tau))$ [1]. as well as by shifts $\Phi(\zeta(s + i\tau), \zeta(s + i\tau, \alpha))$ for some classes of functions $F$ and $\Phi$, where $\zeta(s, \alpha)$ is Hurwitz zeta-function with transcendental parameter $\alpha$. For example, the following statement is true. Let $H(D)$ denote the space of analytic functions on $D$ equipped with the topology of uniform convergence on compacta, and $S = \{g \in H(D) : g^{-1}(s) \in H(D) \quad \text{or} \quad g(s) \not\equiv 0\}$, and for $a_1, ..., a_r \in \mathbb{C}$, $H_r(D) = \{g \in H(D) : (g(s) - a_j)^{-1} \in H(D),\ j = 1, ..., r\}$.

THEOREM 1. *Suppose that $\Phi : H^2(D) \to H(D)$ is a continuous function such that $\Phi(S \times H(D)) = H_r(D)$. If $r = 1$, let $K \subset D$ be a compact subset with connected complement, and let $f(s)$ be a continuous and $\neq a_1$ function on $K$, and analytic in the interior of $K$. For $r \geq 2$, let $K \subset D$ be an arbitrary subset, and $f(s) \in H_r(D)$. Then, for every $\varepsilon > 0$,*

$$\liminf_{T\to\infty} \frac{1}{T}\text{meas}\left\{\tau \in [0, T] : \sup_{s\in K} |\Phi(\zeta(s + i\tau), \zeta(s + i\tau, \alpha)) - f(s)| < \varepsilon\right\} > 0.$$

**REFERENCES**

[1] A. Laurinčikas, Universality of the Riemann zeta-function, J. Number Theory, 130 : 2323–2331, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# INVERSE HEAT TRANSFER PROBLEM FOR THE RECTANGULAR DOMAIN[1]

MARY LENCMANE[1] and ANDRIS BUIKIS[1,2]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: marija.lencmane@lu.lv, buikis@latnet.lv

For practical purposes, e.g. in the building industry, the knowledge of the thermal properties of materials is of great interest. As it is infeasible to directly measure these quantities, they have to be determined by the measurement of related quantities and solving an inverse problem. Measurements can be executed by the Transient Hot Strip (THS), Transient Hot Wire or the Transient Plane source (TPS) methods.

In [1] a transient hot-strip method has been developed for the simultaneously measuring the thermal conductivity and the thermal diffusivity of solids and fluids with low electrical conductivity. The hot strip (the thin metal foil) is used both as a constant plane heat source and a sensor of the temperature increase. By supplying a constant current to the metal strip the output of power is very nearly constant, and by monitoring the subsequent voltage increase over a short period of time after the start of the experiment, it is possible to get precise information on the thermal transport properties of the material surrounding the heat source. In [2] the general theory of the transient plane source (TPS) technique is outlines in some details with approximations for the two experimental arrangements that may be referred to as "hot square" and "hot disk". In [3] the resulting parameter identification was solved in two ways. On the one hand,in idealized case, an analytical approximation of the solution heat conduction equation is used. On the other hand, for more general cases, a new non-linear identification algorithm based on FEM-solution of the heat conduction equation is introduced. In [4] the analytical solution with additional conditions is proposed.

This paper deals with several mathematical three-dimensional formulation of the THS, TPS methods for the different situations: general case with three heat conducting equations with insulating layer, simplified in z direction problem and problem without insulating layer. Problems are reduced to the two-dimension systems and to the systems with non-classical boundary conditions. The main idea of this paper - to find weaker conditions.

**REFERENCES**

[1] S. E. Gustafsson, E. Karawacki, N. Khan. Transient Hot Strip Method for simultaneously measuring thermal conductivity and thermal diffusivity of solids and fluids. *J.Phys.D:Appl.Phys.*, **12:** 1411-1421, 1979.

[2] S. E. Gustafsson. Transient Plane Source technique for thermal conductivity and thermal diffusivity of solids materials. *Rev.Sci.Instrum.*, **62** (3): 797-804, 1991.

[3] R. Model, U. Hammerschmidt. Numerical methods for the determination of the thermal properties by means of transient measurements. *Advanced Computational Methods in Heat Transfer*, **WIT Press, 6:** 407 – 416, 2000.

[4] S. Guseinov, A. Buikis. Inverse heat transport problems for coefficients in two-layer domains and methods for their solution. *Mathematical Modelling and Analysis*, **7** (2): 217 – 228, 2002.

[1] This work is partially supported by the project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund and by the grant 09.1572 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# LARGE-EDDY SIMULATIONS USING VARIOUS SUBGRID SCALE MODELS COMPARED WITH EXPERIMENTAL MEASUREMENTS

ULDIS LOCĀNS[1], NORMUNDS JĒKABSONS[1], RADE MILENKOVIC[2] and SERGEJS DEMENTJEVS[2]

[1] *Ventspils Augstskola*

Inženieru iela 101a, LV-3601, Ventspils, Latvia

[2] *Paul Scherrer Institute*

5232 Villigen PSI, Switzerland

E-mail: uldis.locans@venta.lv

Flow conditions where water flows into rectangular channel trough a nozzle is simulated numerically using open source software OpenFOAM. Simulations are done using Large-Eddy turbulence model with various subgrid scale models in order to achieve solution that best fits experimental measurements. Massive mesh resolution study is done in order to minimize the effect of the computational mesh. Two different geometries where used for large eddy simulations, one without nozzle in model and one where nozzle was included in the model in order to obtain the effects of the nozzle on results. In addition numerical simulations where done in OpenFOAM using k-epsilon turbulence model.

Analysis of the results where done by comparing numerical simulations with experimental measurements. Experimental measurements where done in same geometry as numerical simulations and where set up in Paul Scherrer Institute. Flow measurements in experiment where done using constant temperature anemometer.

**REFERENCES**

[1] Finn E. Jorgensen. *How to measure turbulence with hot-wire anemometers  a practical guide.* , 2002.

[2] Stephen B. Pope. *Turbulent Flows.* Cambridge University, 2000.

[3] J. H. Ferzinger, M. Peric. *Computational Methods for Fluid Dynamics 3rd Edition.* Berlin, 2002.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# BICKEL–ROSENBLATT TEST

AUDRIS LOČMELIS and JĀNIS VALEINIS

*Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: locmelis.audris@gmail.com, valeinis@gmx.net

Bickel and Rosenblatt [1] proposed a goodness-of-fit test based on the integrated squared error between the kernel density estimator and smoothed version of a parametric density estimate. For independent observations they derived the limiting distribution under the null hypothesis and showed the consistency of the test. Later Neumann and Paparoditis [2] showed that the test statistic has the same asymptotic limiting behavior for absolutely regular processes. This result holds true for both simple and composite hypothesis. It makes the test ideal for practical applications, because the test can be used for dependent data without any modifications unlike the classical goodness-of-fit tests, such as Kolmogorov–Smirnov, Pearson and Neyman tests. However, this test has a big drawback – it can not be applied practically due to the bandwidth parameter choice, which is problematic even in the independent case.

Recently for similar test statistics Gao and Gijbels [3] solved the problem of bandwidth choice by establishing closed-form expressions for both size and power functions. In this paper for both independent and weakly dependent observations we investigate the asymptotic behavior of the test, first and second errors by simulation study. Moreover, we show how the result of Gao and Gijbels can be applied for the Bickel–Rosenblatt test statistic.

**REFERENCES**

[1] P. J. Bickel, M. Rosenblatt. On some global measures of the deviation of density function estimators. *The Annals of Statistics*, **1** (6):1071–1095, 1973.

[2] M. H. Neumann, E. Paparoditis. On bootstrapping $L_2$–type statistics. *Statistics & Probability Letters*, **50** (2):137–147, 2000.

[3] J. Gao, I. Gijbels. Bandwidth Selection in Nonparametric Kernel Testing. *Journal of the American Statistical Association*, **103** (484):1584–1594, 2008.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# NUMERICAL SIMULATION OF HEAT DISTRIBUTION IN ELECTRICAL DEVICES

FLORIAN LOOS and HANS-DIETER LIESS

*Universität der Bundeswehr München*

Werner-Heisenberg-Weg 39, 85577 Neubiberg, Germany

E-mail: florian.loos@unibw.de, hdliess@unibw.de

The continuous growth of electrical systems in today's automobiles and the simultaneously associated effect of shortage of available space demand an intelligent energy management with permanent optimization of power generation and power consumption. Electrical devices like cable harnesses, current distribution boxes as junction for electrical connections and conductor rails make an important contribution to the mentioned aspects. In order to keep the heat generation as low as possible, it is necessary to optimize the geometrical composition and the employed material of all components as well as their interaction. Before this optimization can be performed, the single components have to be modelled and simulated by mathematical methods.

For the simulation of these electrical devices, two different approaches to calculate the heat generation are presented. The first one is based on the law of conservation of energy and power balances where we finally derive a nonlinear equation system that is solved by means of a trust region algorithm. The second approach is based on partial differential equations. The main equation governing the heat generation in the devices for the stationary case is the Helmholtz equation:

$$-\lambda \, \Delta \, T = \left( \frac{I}{A} \right)^2 \, \frac{1}{\kappa_0} \left( 1 + \alpha_\rho \left( T - T_{\text{ref}} \right) \right) \quad \text{in} \quad \Omega. \tag{1}$$

The heat transmission from solid material to air is described by a Robin type boundary condition

$$\lambda_{SOL} \frac{\partial T}{\partial n} + \alpha(T) \; T = \alpha(T) \; T_{AIR} \quad \text{on} \quad \Gamma_{INT}, \tag{2}$$

where $\alpha(T)$ denotes the temperature-dependent heat transmission coefficient. It contains convective and radiative effects and causes the system to be nonlinear.

The system of partial differential equations is solved via the finite element method. To treat the appearing nonlinearities, we use the iterative method of Newton-Raphson.

Numerical results of both approaches are compared and pros and cons regarding creation time, calculation time and practical applicability are shown.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# APPROXIMATION OF ANALYTIC FUNCTIONS BY ZETA-FUNCTIONS OF NEWFORMS AND PERIODIC HURWITZ ZETA FUNCTIONS

RENATA MACAITIENĖ

*Šiauliai University*

P. Višinskio str. 19, Šiauliai LT-77156, Lithuania

*Šiauliai State College*

Aušros al. 40, Šiauliai LT-76241, Lithuania

E-mail: renata.macaitiene@mi.su.lt

Let $F(z)$ be a newform of weight $\kappa$ and level $N$ with the Fourier series expansion

$$F(z) = \sum_{m=1}^{\infty} c(m)\mathsf{e}^{2\pi imz}.$$

The zeta-function $\zeta(s, F)$, $s = \sigma + it$, attached to the form $F(z)$ is defined, for $\sigma > \frac{\kappa}{2}$, by

$$\zeta(s, F) = \prod_{p|N} \left(1 - \frac{c(p)}{p^s}\right)^{-1} \prod_{p\nmid N} \left(1 - \frac{c(p)}{p^s} + \frac{1}{p^{2s-1}}\right)^{-1},$$

and is analytically continued to an entire function.

Let $\mathfrak{a} = \{a_m\}$ be a periodic sequence of complex numbers with period $k \in \mathbb{N}$, and $0 < \alpha \leq 1$. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$ is defined, for $\sigma > 1$, by

$$\zeta(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s},$$

and is meromorphically continued to the whole complex plane.

Suppose that the numbers $\alpha_1, ..., \alpha_r$ are algebraically independent over the field of rational numbers, and $\mathfrak{a}_{jl} = \{a_{mjl}\}$, $j = 1, ..., r$, $l = 1, ..., l_j$, are periodic sequences of complex numbers. We prove that, under some additional hypothesis on the coefficients $a_{mjl}$, a collection of analytic functions is approximated uniformly on compact subsets of some region by shifts of the functions $\zeta(s, F)$, $\zeta(s, \alpha_1; \mathfrak{a}_{11}, ..., \zeta(s, \alpha_1; \mathfrak{a}_{1l_1}), ..., \zeta(s, \alpha_r; \mathfrak{a}_{r1}), ...,$
$\zeta(s, \alpha_r; \mathfrak{a}_{rl_r})$. For the proof of the approximation theorem, a probabilistic model is applied.

**REFERENCES**

[1] A. Laurinčikas, K. Matsumoto and J. Steuding. The universality of $L$-functions asocciated with new forms. *Izv. Math.*, **67** 77–90, 2003.

[2] A. Laurinčikas and S. Skerstonaitė. Joint universality for periodic Hurwitz zeta-functions. II. In: *New Directions in Value-Distribution Theory of Zeta and L-functions*, Würzburg conf. 2008, R. Steuding, J. Steuding (Eds.), Shaker Varlag, Aachen, 2009, 161–169.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# COPULA BASED SEMIPARAMETRIC REGRESSIVE MODELS

ANDREJS MATVEJEVS and JEGORS FJODOROVS

*Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

E-mail: andrejmef@yahoo.com

The possibility of identifying nonlinear time series using nonparametric estimates of the conditional mean and conditional variance were studied in many papers (see, for example, [1], and references there). As a rule analyzing the dependence structure of stationary time series $\{x_t, t \in \mathbb{Z}\}$ the authors apply regressive models defined by invariant marginal distributions and copula functions that capture the temporal dependence of the processes. As it indicated in [1] this permits to separate out the temporal dependence (such as tail dependence) from the marginal behavior (such as fat tailedness) of a time series. One more advantage of this type regressive approach is a possibility to apply probabilistic limit theorems for transition from deference equations to continuous time stochastic –differential equations ([2],[3]). In our paper we also study a class of copula-based semiparamric stationary Markov models in a form of scalar difference equation

$$t \in \mathbb{Z}: \quad X_t = X_t + \varepsilon f(X_{t-1}, \varepsilon) + \varepsilon g(X_{t-1}, \varepsilon)\xi_t \tag{1}$$

where $\{\xi_t, t \in \mathbb{Z}\}$ is i.i.d., $N(0,1)$, and $\varepsilon$ is a small positive parameter, which will be used for diffusion approximation of (1). Regressions (1) are high-usage equations for simulation and parameter estimation of stochastic volatility models ([2]). But unfortunately defined by (1) Markov chain has incompact phase space that complicates an application of probabilistic limit theorem. Copula approach helps to simplify asymptotic analysis of (1). Let us remember that to construct a copula $C(u,v)$ for pair $\{X_{t-1}, X_t\}$ from (1) one should find a marginal invariant distribution $F(x)$ for $X_t$ and to substitute this in joint distribution function $H(x,y) = \mathbf{P}(X_{t-1} \le x, X_{t-1} \le y)$, that is, $C(u,v) = H(F^{-1}(u), F^{-1}(v))$ and $H(x,y) = C(F(x), F(y))$. Due to persistence of small parameter $\varepsilon$ after substitution $U_t = F(X_t)$ in equation (1) for further diffusion approximation one can write a difference equation in a same form like (1):

$$t \in \mathbb{Z}: \quad U_t = U_t + \varepsilon \hat{f}(U_{t-1}, \varepsilon) + \varepsilon \hat{g}(U_{t-1}, \varepsilon)\xi_t \tag{2}$$

but now this equation defines Markov chain on the compact $[0,1]$. This makes easier formulae construction for transition probability and further estimators of functions $\hat{f}(u)$ and $\hat{g}(u)$. After diffusion approximation of (2) one can make inverse substitution and derive stochastic differential equation as diffusion approximation for (1).

**REFERENCES**

[1] X.Chen, Y.Fan. Estimation of copula-based semiparametric time series models. *Journal of Econometrics*, **130** 307–335, 2006.

[2] D.B. Nelson. ARCH models as diffusion approximations. *Journal of Econometrics*, **441** 1-2: 7—38, 1990.

[3] Y. Ait-Sahalia, R. Kimmel. Maximum likelihood estimation of stochastic volatility models. *Journal of Financial Economics*, **83** 413–452, 2007.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# MATHEMATICAL MODELS FOR LOW DIMENSIONAL NANOSTRUCTURES: ANALYSIS, NUMERICS AND CONTROL

RODERICK MELNIK

*Department of Mathematics and $M^2NeT$ Laboratory, Wilfrid Laurier University*

75 University Avenue West, Waterloo, ON, Canada N2L 3C5

E-mail: rmelnik@wlu.ca, URL: http://www.m2netlab.wlu.ca

Low-dimensional semiconductor nanostructures (LDSNs) are challenging objects to study from both, fundamental physics and mathematical points of view. These objects are receiving increasing attention as key components of many optoelectronic devices. Quantum dots (QDs), LDSNs in which the motion of electrons is confined from all three spatial dimensions, can also be used as biological tags in DNA analysis, as well as in other bio-technological applications, while the idea of using a spin confined to a QD as a qubit promises imminent breakthrough in quantum information processing. The number of practical applications of LDSNs continues to grow which requires the development of adequate mathematical models for their description and efficient numerical approximations.

Despite a wide range of current and potential applications, properties of QDs are still frequently analyzed with simplified mathematical models, incapable to account correctly for many effects that are coming from other than quantum mechanical scales (e.g., strain, piezoelectric, thermal and other important effects). In this contribution, our main emphasis is on the mathematical models where the coupling between quantum and continuum mechanics parts is essential.

We will focus on coupled and nonlinear effects and their incorporation in new models for the analysis of properties of LDSNs. Based on several new coupled mathematical models, we will demonstrate that *coupled* electro-mechanical effects can lead to pronounced contributions in electronic bandstructure calculations of LDSNs such as quantum dots, wires, and even wells. Some such effects are essentially nonlinear [1]. As thermal effects coupled with electric and mechanical fields in LDSNs are becoming increasingly important, we will also present results on the application of the fully coupled model of thermoelectroelasticity [2] to the analysis of the influence of these effects on the overall properties of LDSNs. The nonlinear mechanics of LDSNs will also be analyzed in some special cases based on coupled models consisting of the time dependent Ginzburg-Landau equation and the conservation balance laws with nonlinear coupling between stress, strain and the phase order parameter. A number of numerical examples will be given to illustrate the theory.

Finally, we will discuss new analytical and numerical modelling techniques [3] to control single electron spin states adiabatically through the application of the geometric Berry phase.

**REFERENCES**

[1] R. Melnik and R. Mahapatra. Coupled effects in quantum dot nanostructures with nonlinear strain and bridging modelling scales. *Computers and Structures*, **85** (11-14): 698-711, 2007.

[2] S. R. Patil and R.V.N. Melnik. Thermoelectromechanical effects in quantum dots. *Nanotechnology*, **20** (12): 125402, 2009.

[3] S. Prabhakar, J. E Raynolds, A. Inomata and R. Melnik. Manipulation of single electron spin in a GaAs quantum dot through the application of geometric phases: the Feynman disentangling technique. *Physical Review B*, **82** 195306, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# UNIQUENESS OF APPROXIMATE SOLUTION IN SPLINE COLLOCATION METHOD FOR BOUNDARY VALUE PROBLEMS

PEETER OJA

*Institute of Mathematics, University of Tartu*

J.Liivi 2, 50409 Tartu, Estonia

E-mail: Peeter.Oja@ut.ee

We discuss the uniqueness of approximate solution in spline collocation method for the boundary value problem

$$y''(x) + p(x)y'(x) + q(x)y(x) = f(x), \quad x \in (a, b),$$

$$y(a) = \alpha, \quad y(b) = \beta.$$

On the uniform mesh $x_i = a + ih$, $i = 0, \ldots, n$, $h = (b - a)/n$, standard quadratic and linear/linear rational splines $S$ of the form

$$S(x) = \frac{a_i + b_i(x - x_{i-1})}{1 + d_i(x - x_{i-1})}, \quad x \in [x_{i-1}, x_i], \quad 1 + d_i(x - x_{i-1}) > 0, \quad i = 1, \ldots, n,$$

and of class $C^1$ are considered and collocation points $(x_{i-1} + x_i)/2$, $i = 1, \ldots, n$, are used. A natural assumption is that the boundary value problem has the unique solution. Note that the linear/linear rational spline is constant or strictly monotone by itself, therefore, it is a reasonable approximate solution only if the exact solution of the boundary value problem has the same property.

The most adequate results with error estimates for the collocation with low order polynomial splines are presented in [2; 3]. Some results about rational spline collocation are announced in [1].

**REFERENCES**

[1] M. Fischer, E. Ideon and P. Oja. Rational spline collocation for boundary value problems. In:. *Abstracts of the 13th International Conference on Mathematical Modelling and Analysis and 3rd International Conference on Approximation Methods and Orthogonal Expansions (MMA2008 & AMOE2008)*, Tartu, Estonia, 2008, 28.

[2] A. K. A. Khalifa nad J. C. Eilbeck. Collocation with quadratic and cubic splines. *IMA J. Numer. Anal.*, **2(1),** 1982, 111 – 121.

[3] P. Oja and A. Reitsekas. On the collocation and subregions methods with quadratic and cubic splines for the boundary value problems. *Proc. Estonian Acad. Sci. Phys. Math.*, **36(2),** 1987, 118 – 128 *(in Russian).*

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# REDUCTION OF COMMUNICATION TIME FOR SPARSE MATRIX-VECTOR MULTIPLICATION

YUSUKE ONOUE* and SEIJI FUJINO**

**Graduate School of Information Science and Electrical Engineering, Kyushu University*
***Research Institute of Information Technology, Kyushu University*

6-10-1, Hakozaki, Higashi-ku, Fukuoka 812-8581 Japan

E-mail: *onoue@zeal.cc.kyushu-u.ac.jp, **fujino@cc.kyushu-u.ac.jp

Iterative methods are often used for solving linear systems. Sparse Matrix-Vector multiplication (thereafter abbreviated as SpMV) used in iterative methods dominates computational cost of solving linear systems. Accordingly it is crucial for us to compute SpMV efficiently. Specially, in parallel SpMV on distributed memory systems, processors should communicate each other, and it takes much time to communicate in almost cases. The conventional communication methods are listed as below[1].

| | |
|---|---|
| **CM1**: Block broadcasting method, | i.e., each process broadcasts a local block. |
| **CM2**: Block point-to-point exchange method, | i.e., processes exchange blocks which contain needed elements through point-to-point communication. |
| **CM3**: Packed point-to-point exchange method, | i.e., processes exchange exactly needed elements only through point-to-point communication. This packed method involves both packing and unpacking, and minimizes amount of communication for additional computation used in packing and unpacking. |

Therefore, in this article, a communication method is proposed to avoid computation for unpacking used in the above **CM3** by means of ordering entries of matrix in advance. Time for ordering is estimated to be negligible, and the proposed technique enables us to reduce communication time of SpMV. Table 1 demonstrates effectiveness of the proposed method. In Table 1, "**CM4**" means the proposed method. We may call it as **CM3 without unpacking**. Moreover, "np" means number of processors, and "ratio" means also ratio of time of each communication method to that of **CM3**. Table 1 clarifies that our **CM4** is estimated as approximately 10% faster than the original **CM3** when number of processors is set as 16.

**Table 1.**
Elapsed time in seconds of SpMV for matrix cage13 on distributed memory systems.

| np | comm. | time | ratio | speedup |
|---|---|---|---|---|
| 1 | - | 20.24 | - | 1.00 |
| 4 | CM1 | 5.01 | 1.04 | 4.04 |
| | CM2 | 4.82 | 1.00 | 4.20 |
| | CM3 | 4.83 | 1.00 | 4.19 |
| | **CM4** | 4.69 | **0.97** | 4.32 |
| 16 | CM1 | 4.37 | 2.68 | 4.63 |
| | CM2 | 3.01 | 1.85 | 6.72 |
| | CM3 | 1.63 | 1.00 | 12.42 |
| | **CM4** | 1.46 | **0.90** | 13.86 |
| 32 | CM1 | 6.45 | 6.20 | 3.14 |
| | CM2 | 2.44 | 2.46 | 8.30 |
| | CM3 | 0.99 | 1.00 | 20.44 |
| | **CM4** | 0.91 | **0.92** | 22.24 |

## REFERENCES

[1] S. Lee, R. Eigenmann. Adaptive runtime tuning of parallel sparse matrix-vector multiplication on distributed memory systems. In: *Proc. of the 22nd Annual Int. Confer. on Supercomputing*, Greece, 2008, 195 – 204.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# AN AGGREGATION APPROACH FOR SOLVING BILEVEL LINEAR PROGRAMMING PROBLEMS[1]

PAVELS ORLOVS and OLGA MONTVIDA

*Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: pavels.orlovs@gmail.com, olgamontvida@yahoo.com

Our talk deals with the following multi-objective linear programming problem with a finite number of linear objective functions and a single set $D$ of decision variables:

$$P_i: \quad y_i(x) = c_{i1}x_1 + c_{i2}x_2 + ... + c_{ik}x_k \longrightarrow min, \quad i = 0, \ldots, n,$$

$$D: \begin{cases} a_{j1}x_1 + a_{j2}x_2 + ... + a_{jk}x_k \leq b_j, \ j = 1, \ldots, m, \\ x_l \geq 0, \ l = 1, \ldots, k, \end{cases}$$

in the case when objectives are divided between two levels in a hierarchy. We suppose that the upper level consists of one problem $P^U = P_0$ and the lower level is multi-objective: $P_i^L = P_i, \ i = 1, \ldots, n$.

To obtain an appropriate compromise solution we start with solutions of the individual problems of minimization and maximization denoted by $y_i^{min}$ and $y_i^{max}$:

$$y_i^{min} = \min_{x \in D} y_i(x), \quad y_i^{max} = \max_{x \in D} y_i(x), \quad i = 0, \ldots, n,$$

and take the membership function of the objective $y_i$ as a fuzzy real number $z_i$ such that $z_i(y_i^{min}) = 1$ and $z_i(y_i^{max}) = 0, \ i = 0, \ldots, n$. This means that the satisfactory level of the objective $y_i$ is characterized with $z_i(y_i(x))$, $x \in D$, $i = 0, \ldots, n$. For our purposes we use the fuzzy real numbers introduced by B.Hutton (order reversing left semi-continuous functions). For example, one can take

$$z_i(t) = \begin{cases} 1, & t < y_i^{min}, \\ \dfrac{t - y_i^{max}}{y_i^{min} - y_i^{max}}, & y_i^{min} \leq t \leq y_i^{max}, \\ 0, & t > y_i^{max}. \end{cases}$$

We describe a method for solving the problem by using an aggregation operator

$$A(z_0(y_0(x)), \ldots, z_n(y_n(x)))$$

and discuss the choice of $A$ and $z_0, z_1, \ldots, z_n$ for adjusting the decision making process between the different levels and also between the objectives of the lower level with the goal to obtain an appropriate balance.

[1]This work has been supported by the European Social Fund within the project Support for Doctoral Studies at University of Latvia.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# PLUG-IN EMPIRICAL LIKELIHOOD METHOD

LEONORA PAHIRKO and JĀNIS VALEINIS

*Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: leonorapahirko@inbox.lv, valeinis@gmx.net

The empirical likelihood (EL) method, introduced by Owen [1; 2] is widely used nowadays. It is a nonparametric procedure since it doesn't require a statistician to pick a parametric family for the data. EL method involves plug-in when estimating function contains *nuisance* parameters which have to be estimated. In a general setting the plug-in EL method has been introduced in the one-sample case by Hjort *et al.* [3]. They derive the limiting asymptotic distribution of the test statistic and analyse applications from different fields, e.g., survival analysis and nonparametric statistics by checking assumptions for each case in their setup.

We aim to support the theoretical results of Hjort *et al.* [3] by extensive simulation study. Moreover, we consider also the two-sample plug-in empirical likelihood already introduced by Valeinis [4] for structural relationship models. More specifically, we look upon latest results and applications for the two-sample case as well as we do an implementation of plug-in EL method in the one and two sample cases.

**REFERENCES**

[1] A. B. Owen. Empirical likelihood ratio confidence intervals for a single functional. *Biometrika*, **75** (2):237–249, 1988.

[2] A. B. Owen. Empirical likelihood ratio confidence regions. *The Annals of Statistics*, **18** (1):90–120, 1990.

[3] N. L. Hjort, I. W. McKeague and I. Van Keilegom. Extending the scope of empirical likelihood. *Annals of Statistics*, **37** (3):1079–1111, 2009.

[4] J. Valeinis. *Confidence bands for structural relationship models*. Phd thesis, Goerg-August-University of Goettingen, Goettingen, 2007.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# USING KALMAN FILTER TO ESTABLISH WHEN LOYALTY PROGRAMS ARE PROFITABLE

DIDIER P. PASCAULT

*Université de Picardie Jules Verne*

avenue des Facultés, FR-80025, Amiens, France

E-mail: dpas@ieee.org

As attracting of a new customer tends to be more expensive then the retention of a current one, it has become a common practice to develop loyalty or retention programs with an objective to retain customers. Different service and goods providers: car dealers, banks or newspaper editors as to mention the most studied cases, keep even only prospective customers on retention programs. Growing such prospective contacts needs tools to decide on whether to keep, reinforce or when to abandon individual retention programs.

In this research, from models of customer behavior [4], the author develops and configures a Kalman filter [2; 3] which should predict a desired behavior of a growing customer. Comparing proposed Kalman filter prediction with customer data triggers alarms that can lead to commercial actions. Either reinforcing retention - loyalty programs or abandoning a contact.

The proposed methodology was developed for a particular service provider and data from testes are cited.

As balancing efforts between retaining valuable and acquisition of new customers while abandoning non-valuable ones was essential in our studied case, the author tries to prove that the Kalman filter is a suitable tool to trigger correct commercial actions.

Finally, in order to validate the presented methodology we test different hypothesis [1].

## REFERENCES

[1] Ch. Gourieroux and A. Monfort. *Statistics and Econometric Models*. Cambridge University Press, Cambridge, 1995.

[2] M. S. Grewal and A. P. Andrews. *Kalman Filtering. Theory and Practice*. Interscience Publishers, New York, 2001.

[3] J. Lluis de la Rosa, R. Mollet, M. Montaner, D. Ruiz and V. Muòoz. Kalman Filters to Generate Customer Behavior Alarms. In: *Proc. of the Conference on Artificial Intelligence Research and Development*, Sant Juli de Lória , Andorra, 2007, C. Angulo and L. Godo (Eds.), IOS Press, Amsterdam. 2007, 416 – 425

[4] W. Reinartz, J. S. Thomas and V. Kumar. Balancing Acquisition and Retention Resources to maximize Customer Profitability. *Journal of Marketing*, **69** (1):63–79, 2005.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SIMPLICIAL LIPSCHITZIAN OPTIMIZATION WITHOUT THE LIPSCHITZ CONSTANT

REMIGIJUS PAULAVIČIUS and JULIUS ŽILINSKAS

*Vilnius University, Institute of Mathematics and Informatics*

Akademijos 4, LT-08663 Vilnius, Lithuania

E-mail: remigijus.paulavicius@vpu.lt, julius.zilinskas@mii.vu.lt

Lipschitz optimization is one of the most deeply investigated subjects of global optimization. It is based on the assumption that the slope of the objective function is bounded. A function $f\colon D \to \mathbb{R}$, $D \subset \mathbb{R}^n$, is said to be Lipschitz-continuous if it satisfies the condition

$$|f(x) - f(y)| \leq L \|x - y\|, \quad \forall x, y \in D, \tag{1}$$

where $L > 0$ is the Lipschitz constant of the function, $D$ is compact and $\|\cdot\|$ denotes the norm.

Lipschitz optimization methods have some very important advantages comparing with other global optimization algorithms. The methods are deterministic and the algorithms provide the same result each time. The methods can guarantee to find an approximation to the solution to a specified accuracy within finite time. Algorithms can provide bounds on how far they are from the optimum function value. They may be used in situation when an analytical description of the objective function is not available.

The main disadvantage of Lipschitz methods is the requirement to provide the Lipschitz constant of the objective function. The methods assume that this is known but in practice this is rarely the case. DIRECT, developed by Jones et al. [1], is a Lipschitz optimization method which eliminates the need to specify the Lipschitz constant. DIRECT derived its name from *di*viding *rec*tangles, one of its main features. At each iteration the algorithm divides hyper-rectangles which are potentially optimal – where the bound can be best depending on the value of unknown Lipschitz constant.

Advantages and disadvantages of simplicial partitions are discussed in [2]. Since a simplex is a polyhedron in $n$-dimensional space with the minimal number of vertices, simplicial partitions are preferable when the values of an objective function at the vertices of partitions are used to compute bounds. The main disadvantage of simplicial partitions is the requirement to cover the hyper-rectangular feasible region by simplices, however there are standard way to do this. Simplicial partitions may be used to vertex triangulate feasible regions of non rectangular shape defined by linear inequality constraints. Linear inequality constraints may be used to avoid symmetries in optimization problems.

In this talk we develop a simplicial version of Lipschitzian optimization without the Lipschitz constant. The efficiency of the proposed algorithm is evaluated experimentally and compared with the results of other algorithms.

**REFERENCES**

[1] D. R. Jones, C. D. Perttunen and B. E. Stuckmann. Lipschitzian optimization without the Lipschitz constant. *Journal of Optimization Theory and Applications*, **79** (1):79–153, 1993.

[2] J. Žilinskas. Branch and bound with simplicial partitions for global optimization. *Mathematical Modelling and Analysis*, **13** (1):145–159, 2008.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# DIFFUSION APPROXIMATION FOR CUMULATIVE ASSET RETURN WITH COMPOUND POISSON VOLATILITY PROCESS

OKSANA PAV LENKO

*Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

E-mail: oksana.pavlenko@rtu.lv

As it has been derived in [1] the simplest continuous time mathematical model of cumulative excess return on the portfolio $S(t)$ may be written as a linear stochastic Ito differential equation

$$dS(t) = c\sigma^2(t)S(t)dt + \sigma(t)S(t)dw(t) \tag{1}$$

where $\sigma^2(t)$ is a volatility process satisfying continuous time GARCH(1,1) stochastic Ito equation. But in reality it happens that there exists so called "a calm-time intervals" defined as the time intervals between two successive excess volatilities (see, foe example, [2]). Analyzing the return and the volatility of the Nikkei 225 index from 1984 to 2002 the authors of paper [2] showed that volatility process can be approximated by compound Poisson process with switching times $\tau_n, n = 0, 1, 2, ...,$ where calm-time intervals are sufficiently short and identically exponentially distributed with intensity $t/\varepsilon$, and $\varepsilon$ is small positive parameter. To take into account the GARCH(1,1) properties of volatility we assume that jumps of the above compound Poisson process is given by equation

$$\sigma^2(\tau_{n+1}) = \sigma^2(\tau_n) + (\omega - \theta\sigma^2(\tau_n)) + \alpha\sigma^2(\tau_n))(\xi_{n+1}^2 - 1) \tag{2}$$

where $\xi_n$ *i.i.d.*, $N(0, 1)$. This stochastic model may be studied by asymptotical averaging procedure because defined by (2) embedded Markov chain has ergodic probability distribution defined by inverse Gamma-distribution [1] $\Gamma\left(1 + \frac{2\theta}{\alpha^2}; \frac{2\omega}{\alpha^2}\right)$. After averaging of drift and diffusion in (1) by invariant measure one can approximate distribution of excess return on the portfolio $S(t)$ by log-normal distribution defined by solution of stochastic Ito differential equation

$$d\hat{S}(t) = \hat{S}\left[\frac{c\omega}{\theta}dt + \nu(\omega, \theta, \alpha)dw(t)\right] \tag{3}$$

where

$$\nu(\omega, \theta, \alpha) = \frac{\left(\frac{2\omega}{\alpha^2}\right)^{\frac{1}{2}\left(\frac{\alpha^2}{\theta}-1\right)+\frac{2\theta}{\alpha^2}} \omega\Gamma\left(\frac{\omega-\alpha^2}{2\theta}\right)}{\theta\Gamma\left(\frac{2\theta}{\alpha^2}\right)}$$

Now one can use the classical Black-Scholes formula for the European call-option and analyze the sensitivity parameters "Greeks".

**REFERENCES**

[1] D.B. Nelson. ARCH models as diffusion approximations.. *Journal of Econometrics*, **441** 1-2: 7—38, 1990.

[2] T. Kaizoji, S. Bornoldt, and Y. Fujiwara. Exponential Laws of a Stock Price Index and a Stochastic Model.. *Physica A316* , 2002.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SOFT NEIGHBORHOOD SETS[1]

BANU PAZAR VAROL[1], ALEKSANDRS ŠOSTAKS[2] and HALIS AYGÜN[1]

[1] *Department of Mathematics, Kocaeli University*

Umuttepe Campus, 41380 Kocaeli, Turkey

[2] *Department of Mathematics, University of Latvia*

Riga LV-1586, Latvia

E-mail: banupazar@kocaeli.edu.tr, sostaks@lanet.lv, halis@kocaeli.edu.tr

The concept of a soft set was introduced in 1999 by D. Molodstov [1], see also [2], as a new approach for modelling uncertaintes. Somewhat reformulating the original definition, by a *soft set over a set* $X$ we call a triple $(M, E, X)$, where $E$ is a set interpreted as *the set of parameters* and the mapping $M : E \to 2^X$ is referred to as the *soft structure on the set* $X$.

We consider soft sets as a category **SOFTS** whose objects are soft sets $(M, E, X)$ and, given two soft sets $(M, E, X)$, $(N, F, Y) \in \mathcal{O}b(\mathbf{SOFTS})$, we take pairs of mappings $\varphi : X \to Y$ and $\psi : E \to F$ such that $\varphi^{\leftarrow} \circ M \geq N \circ \psi$ as morphisms $(\varphi, \psi) : (M, E, X) \to (N, F, Y)$ in the category **SOFTS**. (Here $\varphi^{\rightarrow} : 2^X \to 2^Y$ is the forward powerset operator induced by $\varphi$, that is $\varphi^{\rightarrow}(A) := \varphi(A)$ for each $A \in 2^X$ (see e.g. [5]).)

The concept of a soft set draw attention both of specialists working in the field of pure mathematics as well as researchers in the area of applied mathematics. This interest was provoked in particular by the fact, that the concept of a soft set is well coordinated with such modern mathematical concepts as a fuzzy set and more general, a many-valued set. In the recent years a series of works was published where *soft generalizations* of different mathematical concepts were introduced and studied. In particular, the concepts of fuzzy soft sets [3], fuzzy soft groups [4], fuzzy soft topologies, fuzzy soft rings, etc., were considered by different authors. On the other hand the aim of our research is *not "a soft generalization"* but rather *"a soft interpretation"* of some known mathematical categories. The base for this interpretation is a special type of a soft set over the powerset $2^X$ when the set $X$ is taken as the sets of parameters. In particular, in this talk we characterize topological spaces as follows:

Given a topological *space* $(X, T)$ we interpret it as a soft *set* $(\mathcal{U}, X, 2^X)$ over the powerset $2^X$ where the soft structure $\mathcal{U} : X \to 2^{2^X}$ describes the topology $T$ by a system of neighborhoods of points $x \in X$. Further, we characterize the category of topological spaces **TOP** as a subcategory **NSOFTS** (called the category of neighbourhood soft sets) of the category **SOFTS**

Developing this idea we characterize also some other categories related to topology (crisp and fuzzy) as subcategories of the category **SOFTS** of soft sets.

**REFERENCES**

[1] D. Molodtsov. Soft set theory- First results. *Computers Math. Applicat.* , **37** (4/5):19-31, 1999.

[2] D. Molodtsov. *Theory of Fuzzy Sets*. Editorial URSS , Moscow, 2004. In Russian

[3] P.K. Maji, R. Biswas, A.R. Roy. Fuzzy soft set. *J. Fuzzy Math.*, **9** pp. 589–602, 2001.

[4] A. Aygünoglu, H. Aygün. Introduction to soft fuzzy groups. *Computers& Math. Appl.*, **58** 1279–1286, 2009.

[5] S.E. Rodabaugh. Powerset operator based foundations for point-set lattice-theoretic fuzzy set theories and topologies. *Quaestiones Mathematicae*, **20** 463–530, 1997.

[1] This work is partially supported by the project 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund and by the grant 09.1570 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON THE ONE STATIONARY PROBLEM WITH NONLOCAL BOUNDARY CONDITION DEPENDING ON THREE PARAMETERS

SIGITA PEČIULYTĖ and SIMONA MIKALAUSKAITĖ

*Vytautas Magnus University*

Vileikos 8, LT-44404, Kaunas, Lithuania

E-mail: s.peciulyte@if.vdu.lt, simona.mikalauskaite@fc.vdu.lt

Let us analyze the Sturm-Liouville problem with one classical boundary condition and other nonlocal integral boundary condition depending on three parameters

$$-u'' = \lambda u, \qquad t \in (0,1), \tag{1}$$

$$u(0) = 0, \tag{2}$$

$$u(1) = \gamma \int_{\xi_1}^{\xi_2} u(t)\mathrm{d}t, \tag{3}$$

with the parameters $\gamma \in \mathbb{R}$ and $0 \leqslant \xi_1 < \xi_2 \leqslant 1$. Separate cases of this problem are investigated in [1; 2]. Also some results on the real part of the problem (1)–(3) spectrum are presented in [3].

In this study we present generalized results on the dependence of the Sturm-Liouville problem spectrum on nonlocal boundary condition parameters. We investigate critical points of the characteristic function and generalized eigenfunctions which can exist for these points. Also we present modelling results on the behaviour of the complex eigenvalues when $\gamma \in \mathbb{R}$.

**REFERENCES**

[1] R. Čiupaila, Ž. Jesevičiūtė and M. Sapagovas. On the eigenvalue problem for the one-dimensional differential opreator with nonlocal integral condition. *Nonlinear Analysis: Modelling and Control*, **9** (2).2004 109 – 116

[2] S. Pečiulytė. Investigation and numerical analysis of boundary problems with various types nonlocal conditions *Doctoral dissertation.* Vilnius Gediminas Technical University, 2006.

[3] S. Pečiulytė and A. Štikonas. On the spectrum of the Sturm-Liouville problem with one nonlocal boundary condition. In: *Differential equations and their applications (DETA'2009): dedicated to Professor M. Sapagovas 70th Anniversary* , proceedings of the international conference, September 10-12, 2009 Panevežys.117-122

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SPLINE COLLOCATION FOR VOLTERRA INTEGRAL EQUATIONS WITH SINGULARITIES

ARVET PEDAS

*Institute of Mathematics*

Liivi 2, Tartu 50409, Estonia

E-mail: arvet.pedas@ut.ee

We propose a smoothing technique associated with piecewise polynomial collocation methods for solving linear Volterra integral equations of the second kind with kernels which, in addition to a weak diagonal singularity, may have a weak singularity at the initial point of the interval of integration.

**REFERENCES**

[1] M. Kolk, A. Pedas and G. Vainikko. High-order methods for Volterra integral equations with general weak singularities. *Numer. Funct. Anal. Optim.*, **30** (9-10):1002–1024, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# HIGHER DEGREE FUZZY TRANSFORMS AND APPLICATIONS

IRINA PERFILIEVA and VILÉM NOVÁK

*University of Ostrava, IRAFM*

30. dubna 22, 701 03 Ostrava 1, Czech Republic

E-mail: Irina.Perfilieva,Vilem.Novak@osu.cz

The goal of this paper is to provide a deeper analysis of fuzzy modeling and its contribution to general mathematics. We believe that this will enrich both of these areas and position fuzzy modeling as a classical mathematical discipline. Moreover, we will be able to combine both in order to derive efficient solutions to many practical problems.

In [1], we introduced the notion of fuzzy transform (or F-transform for short), which explains modeling with fuzzy IF-THEN rules as a specific transformation. From this point of view, the F-transform bridges fuzzy modeling and the theory of linear (in particular, integral) transforms.

In this paper, we aim to show that the F-transform can be generalized to the case $F^m$ ($m \geq 0$), where the $F^m$-transform components are polynomials of degree $m$ [3]. We prove that every polynomial component approximates a certain restriction of an original function and that as the degree of the polynomial increases, so does the quality of approximation. We provide a detailed characterization of the $F^1$-transform with linear polynomials as components. Finally, we introduce an inverse $F^m$-transform and show that it approximates the original function on the whole domain. Moreover, we discuss the quality of approximation in terms of the inverse $F^m$-transform in two approximation spaces (specifically, the space of continuous functions and the space $L_1$), where we show a uniform convergence of a sequence of inverse $F^m$-transforms.

Advanced application of the $F$-transform is its application in the analysis and forecasting of time series [2]. It was demonstrated that by means of it, we can well capture trend cycle of the time series and also effectively forecast it using other special soft computing techniques. We use the $F^1$-transform for estimation of smoother trend cycle and its slope. By combining the latter and tools based on the formal theory of evaluative linguistic expressions, we are able to generate automatically linguistic evaluation of the trend of time series in various time slots.

**REFERENCES**

[1] I. Perfilieva. Fuzzy transforms: Theory and Applications. *Fuzzy Sets and Systems*, **157** 993–1023, 2006.

[2] V. Novák, M. Štěpnička, A. Dvořák, I. Perfilieva, V. Pavliska and L. Vavříčková. Analysis of seasonal time series using fuzzy approach. *Int. Journal of General Systems*, **39** 305–328, 2010.

[3] I. Perfilieva, M. Daňková and B. Bede. Towards a Higher Degree F-transform. *Fuzzy Sets and Systems*, **to appear** .

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# MODELLING TAIL DEPENDENCE OF MULTIVARIATE SKEW $t$-DISTRIBUTION

GAIDA PETERE

*Department of Engineering Mathematics, Riga Technical University*

Kaļķu iela 1, Rīga, LV-1658, Latvia

E-mail: gaida@latnet.lv

Let $X$ and $Y$ be two (possibly dependent) random variables with marginal distribution functions $F(x)$ and $G(y)$ which can belong to different classes of probability distributions. Then the joint distribution function $H(x, y)$ can be presented through a co-called copula functions. Copula is defined as a joint distribution of random variables with uniformly distributed marginals. Several classes of copulas have been constructed which have different mathematical properties. One very important property of a copula is tail dependence. Will a big loss of $X$ enlarge the probability that $Y$ also will be big? Mathematically answer to this question is characterized by the limit of the conditional probability $P(Y|X)$ when value of $X$ tends to infinity, $Y$ is greater or equal to $X$. The limit is the characteristic of tail dependence. It has been proved that for the Gaussian copula which is constructed from multivariate normal distribution, this limit equals to zero. This property makes Gaussian copula not suitable for many financial applications. In the paper of Kollo and Pettere [1] skew $t$-copula is constructed. The construction is based on the multivariate skew $t$-distribution. First applications have shown that the distribution can successfully be used in practice. However, we have not been able to get an analytic expression for the tail dependence. To get some idea about the tail behavior of the copula we have carried out a simulation experiment. The results of simulation will be presented in the talk.

**REFERENCES**

[1] T. Kollo and G. Petere. Parameter Estimation and Application of the Multivariate Skew $t$-Copula. In: *Proceedings of the Workshop held in Warsaw, 25-26 September 2009*, Copula Theory and Its Applications, Springer, 2010, 289 – 298.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# A MONOTONIC SCHEME OF FOURTH ORDER APPROXIMATION FOR THE 2D STEADY-STATE CONVECTION-DIFFUSION EQUATION

VIKTOR POLEVIKOV

*Belarusian State University*

Independence avenue 4, Minsk 220030, Belarus

E-mail: polevikov@bsu.by

When solving many applied convective transfer problems, transition to high Rayleigh number values which are characterized by formation of regions with large temperature and velocity gradients is unavoidable. This is also true for the concentration of suspended particles in the problems of particle diffusion in colloidal suspensions, e.g. in the diffusion process of ferromagnetic particles in a magnetic fluid under the action of a sufficiently strong magnetic field [1]. This makes high demands on stability and approximation properties of a difference scheme. One way of problem solving is to increase the scheme approximation order although it is not easily to satisfy the conflicting demands of high stability and accuracy.

This abstract deals with the 2D steady-state convection-diffusion equation

$$\nabla \cdot (k \nabla u) - \mathbf{v} \cdot \nabla u - qu + f = 0 \tag{1}$$

where $\mathbf{v} = (v_1, v_2)$ is the fluid velocity vector; $u$ is the unknown function and $k > 0$, $q \geq 0$, $f$, $v_1$, $v_2$ are the given sufficiently smooth functions of the space coordinates $x_1$, $x_2$.

Equation (1) is approximated on the uniform space mesh with the step sizes $h_1$ and $h_2$ in $x_1$ and $x_2$ direction, respectively. The difference scheme is constructed on the minimal nine-point pattern. It is of the fourth-order approximation and satisfies the principle of maximum at the step-to-step ratio $1/\sqrt{5} \leq h_1/h_2 \leq \sqrt{5}$. Monotonicity of the scheme is achieved by means of two regularization parameters introduced into the difference operator.

The scheme is tested on known problems of natural convection within a square cavity which either is heated uniformly from the side or with sinusoidal temperature distribution on the upper wall [2]. It is found that the constructed scheme is most effective and has considerable advantages over first and second order schemes under the conditions of developed laminar convection.

**REFERENCES**

[1] V. Polevikov and L. Tobiska. On the solution of the steady-state diffusion problem for ferromagnetic particles in a magnetic fluid. *Mathematical Modelling and Analysis*, **13** (2): 233 – 240, 2008.

[2] B. Berkovsky and V. Polevikov. *Computational experiment in convection.* Universitetskoe, Minsk, 1988. (in Russian)

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# JOINT UNIVERSALITY FOR ZETA-FUNCTIONS OF CUSP FORMS AND PERIODIC HURWITZ ZETA-FUNCTIONS

S. RAČKAUSKIENĖ and DARIUS ŠIAUČIŪNAS

*Šiauliai University*

P. Višinskio 19, LT-77156 Šiauliai, Lithuania

E-mail: santa.rackauskiene@gmail.com, siauciunas@fm.su.lt

Let $F(z)$ be a normalized Hecke eigen cusp form of weight $\kappa$ for the full modular group with the Fourier series expansion

$$F(z) = \sum_{m=1}^{\infty} c(m)\mathrm{e}^{2\pi imz}, \quad c(1) = 1.$$

The zeta-function $\zeta(s, F)$, $s = \sigma + it$, of the form $F(z)$ is defined, for $\sigma > \frac{\kappa+1}{2}$, by

$$\zeta(s, F) = \sum_{m=1}^{\infty} \frac{c(m)}{m^s},$$

and is analytically continued to an entire function. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$, for $\sigma > 1$, is given by Dirichlet series

$$\zeta(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s},$$

and by analytic continuation elsewhere. Here $\alpha$, $0 < \alpha \le 1$, is a fixed parameter, and $\mathfrak{a} = \{a_m\}$ is a periodic sequence of complex numbers.

The report is devoted to the joint universality of the functions $\zeta(s, F), \zeta(s, \alpha_1; \mathfrak{a}_{11}), \dots, \zeta(s, \alpha_1; \mathfrak{a}_{1l_1}), \dots, \zeta(s, \alpha_r; \mathfrak{a}_{r1}), \dots, \zeta(s, \alpha_r; \mathfrak{a}_{rl_r})$. By shifts of these functions, a collection of analytic functions is approximated uniformly on compact subsets of the strips $\{s \in \mathbb{C} : \frac{\kappa}{2} < \sigma < \frac{\kappa+1}{2}\}$ and $\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\}$. For the proof of a joint universality theorem, the method of probabilistic limit theorems in the space of analytic functions is used.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# DISCRETE UNIVERSALITY OF THE RIEMANN ZETA-FUNCTION

J. RAŠYTĖ

*Vilnius University Institute of Mathematics and Computer Science*

Naugarduko 24, LT-03225, Vilnius, Lithuania

E-mail: jovita.ras@gmail.com

It is well known that the Riemann zeta-function $\zeta(s)$, $s = \sigma + it$, is universal in the sense that its shifts $\zeta(s + i\tau)$ or $\zeta(s + imh)$, $\tau \in \mathbb{R}$, $h > 0$, $m \in \mathbb{N} \cup \{0\}$, approximate any analytic function uniformly on compact subsets of some region. In the case of shifts $\zeta(s + imh)$, the above property is called a discrete universality of $\zeta(s)$.

In the report, we consider the discrete universality of composite functions $F(\zeta(s))$. Let $D = \{s \in \mathbb{C}\colon \frac{1}{2} < \sigma < 1\}$, and $H(D)$ denote the space of analytic functions on $D$ equipped with the topology of uniform convergence on compacta. Among other results, we have the following theorem. For $a_1, \dots, a_r \in \mathbb{C}$ and $C \in \mathbb{C}$, let $H_{C;a_1,\dots,a_r}(D) = \{g \in H(D)\colon (g(s) - a_j)^{-1} \in H(D),\ j = 1, \dots, r, \text{ or } g(s) \equiv C \text{ if } \exists a_j = C\}$.

THEOREM 1. *Suppose that the number* $\exp\left\{\frac{2\pi k}{h}\right\}$ *is irrational for all* $k \in \mathbb{Z} \setminus \{0\}$, *and that* $F : H(D) \to H(D)$ *is a continuous function such that* $F(S) = H_{F(0);a_1,\dots,a_r}(D)$. *For* $r = 1$, *let* $K \subset D$ *be a compact subset with connected complement, and* $f(s)$ *be a continuous and* $\neq a_1$ *function on* $K$, *and analytic in the interior of* $K$. *For* $r \geq 2$, *let* $K \subset D$ *be a compact subset, and* $f(s) \in H_{F(0);a_1,\dots,a_r}(D)$. *Then, for every* $\varepsilon > 0$,

$$\liminf_{N\to\infty} \frac{1}{N+1} \sharp\{0 \leq m \leq N\colon \sup_{s\in K} |F(\zeta(s + imh) - f(s))| < \varepsilon\} > 0.$$

If $r = 1$ and $a_1 = 0$, then we have the universality of $e^{\zeta(s)}$ and $\zeta^N(s)$, $N \in \mathbb{N}$. If $r = 2$ and $a_1 = -1$, $a_2 = 1$, then the universality of $\sin \zeta(s)$ follows.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# EFFICIENT NUMERICAL TECHNIQUES FOR STIFF DELAY DIFFERENTIAL MODELS IN BIOSCIENCES

FATHALLA A. RIHAN

*Department of Mathematical Sciences,*

Faculty of Science, United Arab Emirates University, Al Ain, UAE

E-mail: frihan@uaeu.ac.ae

To provide realistic mathematical models for problems with time-lag or after-effect, we should consider retarded functional differential equations (RFDEs), in place of ordinary differential equations (ODEs), such as:

$$y'(t) = f\left(t, y(t), y(\alpha(t, y(t))), \int_{-\infty}^{t} \mathcal{K}(t, s, y(t), y(s))ds\right), \quad t \geq t_0, \tag{1}$$

where $\alpha(t, y(t)) \leq t$ and $y(t) = \psi(t),\ t \leq t_0$. Such retarded equations form a class of equations which is, in some sense, between ODEs and time-dependent partial differential equations (PDEs). These equations generate infinite-dimensional dynamical systems. RFDEs (1) where the integral term is absent are usually called delay differential equations (DDEs) and they assume forms such as

$$y'(t) = f\left(t, y(t), y(\alpha(t, y(t)))\right), \quad \alpha(t, y(t)) \leq t. \tag{2}$$

In this paper, we show how *delay differential models* have a richer mathematical framework and better consistency (compared with models without memory or after-effects) when modeling biosciences phenomena. Basic mathematical models in epidemiology of infectious diseases and interactions between the tumor and immune cells are described to show the consistency of the delay differential models with these real phenomena. However, most of these problems (models) are stiff and reliable computational techniques are then required. In this paper, we provide a class of semi-implicit Continuous Rune-Kutta methods, such as mono-implicit Runge-Kutta Scheme, for delay differential equations. Some examples are considered to show the efficiency of the numerical methods for both stiff and non-stiff problems.

**REFERENCES**

[1] F.A. Rihan, Delay Differential Models in Dynamic Diseases, The Proceedings of the International Conference on Bioinformatics and Computational Biology 2010, Hawaii, USA, March 24-26 (2010), pp 73–79.

[2] F.A. Rihan, E.H. Doha, M.I. Hassan and N.M. Kamel, Numerical Treatments for Volterra Delay Integro-Differential Equations, *Comput. Methods Appl. Math.*, **9**(3) (2009) 292–308.

[3] C.T.H Baker, G. Bocharov, E. Parmuzin, F.A. Rihan, Some aspects of causal & neutral equations used in modelling, *J. Comput. Appl. Math.* **229** (2009) 335–349.

[4] G.A. Bocharov & F.A. Rihan, Numerical modelling in biosciences using delay differential equations, *J. Comput. Appl. Math.* **125** (2000) 183–199.

[5] F.A. Rihan, *Numerical Treatment of Delay Differential Equations in Bioscience*, Ph.D. thesis, Department of Mathematics, University of Manchester, U.K., 2000.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# OPTIMIZATION THE STRUCTURE OF MULTI-LEVEL SELECTION PROCEDURE

TIIT RIISMAA

*Institute of Cybernetics at Tallinn University of Technology*

Akademia tee 21, Tallinn 12618, Estonia

E-mail: tiitr@ioc.ee

A method of description and optimization of the structure of multi-level parallel and distributed processing systems is presented. Consider all s-levels hierarchies, where nodes on level i are selected from the given nonempty and disjoint sets and all selected nodes are connected with selected nodes on adjacent levels. All oriented trees of this kind form the feasible set of hierarchies. Suppose $m_i \times m_{i-1}$ matrix $Y_i = (y^i_{jr})$ is an adjacent matrix of levels $i$ and $i-1 (i = 1, ..., s)$.

**Theorem.** All hierarchies with adjacent matrixes $Y_1, ..., Y_s$ from the described set of hierarchies satisfy the condition

$$Y_s \cdot ... \cdot Y_1 = \underbrace{(1, ..., 1)}_{m_0}. \tag{1}$$

The general optimization problem is stated as a problem of selecting the feasible structure which corresponds to the minimum of total loss given in the separable-additive form:

$$\min\{\sum_{i=1}^{s}\sum_{j=1}^{m_i} h_{ij}(\sum_{r=1}^{m_{i-1}} d^i_{jr} y^i_{jr}) \ \mid \ Y_s \cdot ... \cdot Y_1 = \underbrace{(1, ..., 1)}_{m_0}\} \ \textit{over} \ \ Y_1, ..., Y_s. \tag{2}$$

Here $h_{ij}(\cdot)$ is an increasing loss function of j-th elememt on i-th level and $d^i_{jr}$ is the element of $m_i \times m_{i-1}$ matrix $D_i$ for the cost of connection between the i-th and (i-1)-th level. The meaning of functions $h_{ij}(k)$ depends on the type of particular system. A recursive algorithm is constructed to solve this general problem of optimal multi-level selection procedure. If

$$d^i_{jr} = d^i_j (i = 1, ..., s; j = 1, ..., m_i; r = 1, ..., m_{i-1}), h_{ij}(k) = h_i(k) (i = 1, ..., s; j = 1, ..., m_i),$$

the general problem (2) transforms into the mutually dependent phases:

$$\min\{\sum_{i=1}^{s} g_i(p_{i-1}, p_i) \ \mid \ p_0 \geq p_1 \geq ... \geq p_s, p_s = 1\} \ \textit{over} \ \ \textit{integer} \ \ p_1, ..., p_{s-1}, \tag{3}$$

$$g_i(p_{i-1}, p_i) = \min\{\sum_{j=1}^{p_i} h_{ij}(k_{ij}) \ \mid \ \sum_{j=1}^{p_i} k_{ij} = p_{i-1}\} \ \textit{over} \ \ \textit{integer} \ \ k_{i1}, ..., k_{ip_i}, \tag{4}$$

Free variables of the inner minimization (4) are used to describe the connections between the adjacent levels. Free variables of the outer minimization (3) are used for the representation of the number of elements at each level. Two different classes of iteration methods are developed. For solving the reduced problem the recursive algorithm is constructed, where index of level is the index of recursion. Also a numerical method of local searching is developed.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# JOINT VALUE-DISTRIBUTION OF PERIODIC HURWITZ ZETA-FUNCTION

AUDRONĖ RIMKEVIČIENĖ

*Šiauliai College*

Aušros ave. 40, LT-76241 Šiauliai, Lithuania

E-mail: audronerim@gmail.com

Let $\mathfrak{a} = \{a_m : m \in \mathbb{N} \cup \{0\}\}$ be a periodic sequence of complex numbers $\alpha$, where $0 < \alpha \leq 1$, be a fixed parameter, and let $s = \sigma + it$ be a complex variable. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$ is defined, for $\sigma > 1$, by the series

$$\zeta(s, \alpha; \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m+\alpha)^s},$$

and by analytic continuation elsewhere. In the report, we consider joint limit theorems for a collection of periodic Hurwitz zeta-functions $\zeta(s, \alpha_1; \mathfrak{a}_1), ..., \zeta(s, \alpha_r; \mathfrak{a}_r)$. The cases of algebraically independent and rational parameters $\alpha_1, ..., \alpha_r$ are disused. More precisely, we prove the following statement. Suppose that $\min_{1 \leq j \leq r} \sigma_j > \frac{1}{2}$. Then we obtain that the probability measure

$$\frac{1}{T}\mathrm{meas}\big\{t \in [0, T] : \big(\zeta(\sigma_1 + it, \alpha_1; \mathfrak{a}_1), ..., \zeta(\sigma_r + it, \alpha_r; \mathfrak{a}_r)\big) \in A\big\}, \quad A \in \mathcal{B}(\mathbb{C}^r),$$

converges weakly to the explicitly given probability measure on $(\mathbb{C}^r, \mathcal{B}(\mathbb{C}^r))$ as $T \to \infty$. Here $\mathrm{meas}\{A\}$ denotes the Lebesgue measure of a measurable set $A \subset \mathbb{R}$, and $\mathcal{B}(\mathbb{C}^r)$ is the class of Borel sets of the space $\mathbb{C}^r$. The obtained result continues the investigations of [1].

**REFERENCES**

[1] A. Rimkevičienė. Limit theorems for the periodic Hurwitz zeta-function. *Šiauliai Math. Semin.*, **5 (13)** : 55–69, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# DESIGN OF GEOMETRIC MULTIGRID METHODS FOR QUADRATIC FEM DISCRETIZATIONS ON SEMI-STRUCTURED GRIDS

C. RODRIGO, F.J. GASPAR and F.J. LISBONA

*Centro Universitario de la Defensa*
Academia General Militar
Ctra. Huesca s/n
50090 Zaragoza
Spain

E-mail: carmenr@unizar.es

High order finite element methods are commonly used in practical computations due to their improved approximation properties in comparison to the linear case. This motivates the study of fast solvers for linear systems arising from these type of discretizations.
On the other hand, geometric multigrid methods are among the most powerful techniques to solve sparse linear systems of equations, and it has been proved that they can be successfully combined with semi-structured grids in order to solve problems on relatively complex domains, see for example [1].
Moreover, to design geometric multigrid methods, Local Fourier Analysis (LFA) is a very useful tool. However, LFA for quadratic finite element discretizations can not be performed in a standard way, since the discrete operator is defined by different stencils depending on the location of the points in the grid. In this work, a multicolor local Fourier analysis is presented to analyze multigrid solvers for quadratic finite element discretizations on structured grids. This analysis is later used to choose suitable components on each triangular block of a unstructured partition of the domain in order to design an efficient multigrid solver based on an hybrid smoother combining the choices on the different coarse triangles. Some results showing the good correspondence between the two-grid convergence factors predicted by the analysis and the experimentally computed asymptotic convergence factors are presented. Also, several experiments illustrating the application of this analysis to the design of multigrid methods on semi-structured grids are developed in this talk.

## REFERENCES

[1] F.J. Gaspar, F.J. Lisbona, F.J. Gracia and C. Rodrigo. Multigrid finite element methods on semi-structured triangular grids for planar elasticity. *Numerical Linear Algebra with Applications*, **17** 473–493, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# GREEN'S FUNCTION FOR DISCRETE SECOND-ORDER PROBLEMS WITH NONLOCAL BOUNDARY CONDITIONS

SVETLANA ROMAN and ARTURAS ŠTIKONAS

*Institute of Informatics and Mathematics, Vilnius University*

Akademijos 4, LT-08663 Vilnius, Lithuania

E-mail: svetlana.roman@mii.vu.lt; arturas.stikonas@mif.vu.lt

We analyze the second order nonhomogeneous difference equation with two additional conditions

$$\mathcal{L}u := a_i^2 u_{i+2} + a_i^1 u_{i+1} + a_i^0 u_i = f_i, \quad i = 0, 1, \dots, n-2, \tag{1}$$

$$\langle L_1, u\rangle = g_1 \in \mathbb{K}, \qquad \langle L_2, u\rangle = g_2 \in \mathbb{K}, \tag{2}$$

where $a^2, a^0 \neq 0$ and $L_1, L_2$ are linear independent functionals.

Let $\boldsymbol{u} = \{u^1, u^2\}$ be a fundamental system of homogeneous equation (1), $\boldsymbol{L} = [L_1, L_2]$ and

$$D(\boldsymbol{L})[\boldsymbol{u}] = \begin{vmatrix} \langle L_1, u^1\rangle & \langle L_2, u^1\rangle \\ \langle L_1, u^2\rangle & \langle L_2, u^2\rangle \end{vmatrix}, \quad D(\boldsymbol{L}, \delta_i)[\boldsymbol{u}, u^0] = \begin{vmatrix} \langle L_1, u^0\rangle & \langle L_2, u^0\rangle & u_i^0 \\ \langle L_1, u^1\rangle & \langle L_2, u^1\rangle & u_i^1 \\ \langle L_1, u^2\rangle & \langle L_2, u^2\rangle & u_i^2 \end{vmatrix}.$$

We find an expression for the solution and construct Green's function for this problem. Formula

$$\begin{aligned} G_{ij} &= G_{ij}^{\mathrm{cl}} - \langle L_1^k, G_{kj}^{\mathrm{cl}}\rangle \frac{D(\delta_i, L_2)}{D(\boldsymbol{L})} - \langle L_2^k, G_{kj}^{\mathrm{cl}}\rangle \frac{D(L_1, \delta_i)}{D(\boldsymbol{L})} \\ &= \frac{1}{D(\boldsymbol{L})[\boldsymbol{u}]} \begin{vmatrix} \langle L_1, u^1\rangle & \langle L_2, u^1\rangle & u_i^1 \\ \langle L_1, u^2\rangle & \langle L_2, u^2\rangle & u_i^2 \\ \langle L_1^k, G_{kj}^{\mathrm{cl}}\rangle & \langle L_2^k, G_{kj}^u\rangle & G_{ij}^u \end{vmatrix} = \frac{D(\boldsymbol{L}, \delta_i)[\boldsymbol{u}, G_{\cdot,j}^u]}{D(\boldsymbol{L})[\boldsymbol{u}]}. \end{aligned}$$

is valid. It allows to find Green's function for equation with additional conditions if we know Green's function $G^{\mathrm{cl}}$ for the same equation but with another additional conditions. We apply formulae to Green's functions for problems with nonlocal boundary conditions.

We can generalized these results for $n$-order difference equation.

**REFERENCES**

[1] S.K. Ntouyas. Nonlocal initial and boundary value problems: a survey. In: *Handbook of Differential Equations. Ordinary Differential Equations, vol. 2,* A. Cañada, P. Drábek and A. Fonda (Eds.), Elsevier, North Holland, 2005, 461–558.

[2] S. Roman and A. Štikonas. Green's function for discrete second-order problems with nonlocal boundary conditions. *Boundary Value Problems*, **2011:** 1–23, 2011. doi:10.1155/2011/767024.

[3] A. Štikonas and S. Roman. Stationary Problems with Two Additional Conditions and Formulae for Green's Functions. *Numer. Funct. Anal. Optim.*, **30** (9):1125–1144, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# DISCRETIZATION METHODS FOR THE VARIABLE MODEL ORDER ESTIMATION

M. ROMANOVAS[1], L. KLINGBEIL[1], M. TRÄCHTLER[1] and Y. MANOLI[1,2]

[1] *HSG-IMIT – Institute of Microsystems and Information Technology*

Wilhelm-Schickard-Straße 10, D-78052, Villingen-Schwenningen, Germany

E-mail: (michailas.romanovas, lasse.klingbeil, martin.traechtler)@hsg-imit.de

[2] *Fritz Huettinger Chair of Microelectronics, Department of Microsystems Engineering (IMTEK), University of Freiburg*

Georges-Köhler-Allee 101, D-79110, Freiburg, Germany

E-mail: manoli@imtek.de

The work discusses several approximate discretization schemes suitable for the realization of fractional-order differentiator within the estimation algorithms. The presented rational approximation schemes are believed to converge more rapidly compared to polynomial approximations such as a direct discretization using the finite memory length expansion of Grünwald-Letnikov definition [1; 2]. A practial usefulness of the schemes is assessed by combining the discretization schemes with explicit model order estimation for a benchmark problem of varying model order. The explicit model order tracking is based on Unscented Kalman filter (UKF), where the model order is estimated as a part of the filter state and the nonlinearity of the order estimation is addressed by approximating the probability distribution with determenistically sampled reduced set of so-called $\sigma$-points. Differently from a conventional Extended Kalman filtering, the presented method employ original model prediction and measurement functions to relate approximation coefficients to the order of the model and, therefore, avoid calculation of associated potentially very complex Jacobians for covariance propagation [3].

Comparative remarks between the presented approximation schemes are drawn with respect the accuracy of the discussed schemes, length of approximation history and suitability of the methods for a particular recursive Bayesian estimation technique.

**REFERENCES**

[1] B. Vinagre, I. Podlubny, A. Hernandez and V. Feliu. Some approximations of fractional order operators used in control theory and applications. *Fractional Calculus Appl. Anal.*, **3** (3):231-248, 2000.

[2] M. Romanovas, L. Klingbeil, M. Traechtler and Y. Manoli. Fractional model based Kalman filters for angular rate estimation in vestibular systems. In: *Proc. of the IEEE 10th International Conference on Signal Processing (ICSP), Beijing, China*, 2010, pp. 1 – 4.

[3] M. Romanovas, L. Klingbeil, M. Traechtler and Y. Manoli. Implicit Fractional Model Order Estimation Using Interacting Multiple Model Kalman Filter. In: *18th IFAC World Congress, Milano, Italy*, accepted, August 2011.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# FLUID STRUCTURE INTERACTION COMPUTATIONS FOR LANN WING BASE ON NONLINEAR PROPERTIES OF STRUCTURE

R.ROSZAK, W. STANKIEWICZ and M. MORZYŃSKI

*Institute of Combustion Engines and Transport, Poznan University of Technology*

ul. Piotrowo 3, 60-965 Poznan, Poland

E-mail: robert.roszak@put.poznan.pl

The aim of the work is to develop and investigate uid and structure codes coupling into one aeroelastic code based full scale flow model computation and and nonlinearity of structural model base on LANN Wing case. This phenomenon has important influence in many aeronautical applications. Aerodynamic forces from stream of air lead to deformations of the structure and these deformations change aerodynamic forces. More complicated dynamic analysis algorithm, e.g. including hyper-elastic materials, is also supported. The existing aeroelastic tool, based on high-delity ow solver, was successfully tested and the results were compared with experimental data for LANN Wing test case. The computations were carried out in parallel environment for uid mesh larger than one million tetra elements. Structural part of in-house aeroelastic code includes nonlinear material model - Mooney Rivlin. Structural model was tested for linear and nonlinear case. Reduced Order Model of the ow for aeroelastic analyses with geometrical nonlinaerities, based on Galerkin method, is under development. It requires mode basis, computed in high-delity com- putations (like RANS or Direct Numerical Simulation of Navier-Stokes or Euler equations), to approximate the high-delity solution and in the projection of the governing equations. In the paper the results of the unsteady numerical simulations for LANN wing (as an example of applying nonlinear material models) are presented. Next, the Proper Orthogonal Decomposition of resulting velocity and pressure elds is performed to create mode basis that is required for further Reduced Order Modelling.

**REFERENCES**

[1] C. Farhat, M. Lesoinne, P. Stern and S. Lanteri. High performance solution of three-dimensional nonlinear aeroelastic problems via parallel partitioned algorithms: methodology and preliminary results. *Advances in Engineering Software*, **28** (1):43–61, 1997.

[2] D. Szelag, Z. Lorenc, L. Fornasier and P. Posadzy. *Difference methods for initial value problems*. Proceedings of the International Forum on Aeroelasticity and Structural Dynamics, Munich, Germany, 2004.

[3] L. Liu, E.H. Dowell and J.P. Thomas. A high dimensional harmonic balance approach for an aeroelastic airfoil with cubic restoring forces.. *Journal of Fluids and Structures*, **23** (3):351–363, 2007.

[4] B.R. Noack, K. Afanasiev, M. Morzyski, G. Tadmor and F. Thiele. A hierarchy of low-dimensional models for the transient and post-transient cylinder wake. Journal of Fluids and Structures. *Journal Fluid Mechanics*, **497** 335–363, 2003.

[5] R. Roszak, W. Stankiewicz and M. Morzyński. Article in proceedings. In: *Journal of Mathematical Modelling and Analysis*, Preliminary Aeroelastic Computations for Flow Model Reduction and Nonlinear Properties of Structure, 2010. (in Lithuania)

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# AN ANALYSIS OF APPROXIMATION ON AN L-FUZZY SET BASED ON THE L-FUZZY VALUED INTEGRAL[1]

VECISLAVS RUZA[1] and SVETLANA ASMUSS[1,2]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: ruza.vecislavs@gmail.com, svetlana.asmuss@lu.lv

In order to estimate the quality of approximation on an L-fuzzy set, we need an appropriate L-fuzzy analogue of a norm. In this talk we apply the L-fuzzy integral introduced in our previous papers to investigation of the error of approximation of a real valued function $f$ on an L-fuzzy set $E$.

We assume that $L$ is a completely distributive lattice, operations with L-fuzzy sets and L-fuzzy real numbers are based on the minimum t-norm, $f$ is measurable with respect to a finite measure $\nu$ defined on a $\sigma$-algebra $\Phi$ of crisp sets, $\mu$ is the t-norm based extension of $\nu$ to an L-fuzzy valued measure on a tribe $\Sigma$ of L-fuzzy sets and $E$ is measurable with respect to $\mu$, i.e. $E \in \Sigma$. The t-norm based construction of an L-fuzzy valued measure and L-fuzzy valued integral was considered in [1; 2].

Taking as a basis our previous works now we introduce an L-fuzzy valued norm defined by the L-fuzzy valued integral and describe the space $\mathscr{L}_1(E, \Sigma, \mu)$ of L-fuzzy integrable over $E \in \Sigma$ real valued functions. Notice that the norm $\|f\|_\mu$ in this case is characterized by an L-fuzzy real number, i.e. an order reversing left semi-continuous function taking values in $L$ (for our purposes we use the fuzzy real line introduced by B.Hutton).

We show a possible application of the L-fuzzy valued norm described above in approximation theory. Being more precise, we use it to estimate on $E$ the error of approximation $\mathscr{A}$ of a function $f \in \mathscr{L}_1(E, \Sigma, \mu)$:

$$e(f, \mathscr{A}, E) = \|f - \mathscr{A}f\|_\mu.$$

By a method of approximation we mean any operator

$$\mathscr{A} \colon \mathscr{L}_1(E, \Sigma, \mu) \to \mathscr{U},$$

where $\mathscr{U} \subset \mathscr{L}_1(suppE, \Phi, \nu)$ is a finite-dimensional space of functions used for approximation (it could be a space of polynomials or splines). Finally, we discuss the results of such analysis of approximation for some numerical examples.

## REFERENCES

[1] V. Ruzha, S. Asmuss, A construction of an L-fuzzy valued measure of L-fuzzy sets. *Proceedings of the IFSA-EUSFLAT 2009 Conference* (IFSA-EUSFLAT 2009), pages 1735–1739, Lisbon (Portugal), 2009.

[2] V. Ruza, On an L-fuzzy valued integral with respect to an L-fuzzy valued measure. *Proceedings of the* $10^{th}$ *APLIMAT 2011 Conference* (APLIMAT 2011), pages 605–614, Bratislava (Slovakia), 2011.

[1] This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and 2009/0138/1DP/1.1.2.1.2/09/IPIA/VIAA/004 of the European Social Fund and by the grant 09.1570 of the Latvian Council of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# THERMAL FACEPRINT AND 3D GEOMETRIC FEATURES OF HAND FOR BIOMETRIC IDENTITY VERIFICATION[1]

MICHAŁ RYCHLIK, WITOLD STANKIEWICZ and MAREK MORZYŃSKI

*Division of Methods of Machine Design, Poznan University of Technology*

Piotrowo 3, 60-965 Poznań, Poland

E-mail: rychlik.michal@poczta.fm

The person identification and verification has permanently increasing importance, in many different industrial sectors as well as in everyday human activity,. As a result of such situation, the new technologies and methods of security systems are developed [1; 2]. One of the main directions of research works are the biometric systems.

In this work we present two different types of features for pattern recognition. The first set of data consists of 2D infrared images of faces in order to create the unique thermal faceprint. Second set of data consists of 3D hand shapes to classify the individual volume features.

For extraction of the recognition features in both cases the Principal Component Analysis is used. PCA decomposes the population of input objects (2D images or 3D surfaces) into mean geometry and individual features (empirical modes), describing deviations from mean value.

Each original object $S_i$ is reconstructed by using some $K$ principal components:

$$S_i = \bar{S} + \sum_{k=1}^{K} a_{ki}\Psi_k, \qquad i = 1, 2, \cdots, M \tag{1}$$

where $\bar{S}$ is a mean shape, $\Psi_k$ is an eigenvector representing the orthogonal mode (the feature computed from data base), $a_{ki}$ is coefficient of eigenvector $\Psi_k$ for $i$-th object. For both (infrared face images and hand shapes) sets of input data, the PCA decompositions were done. The mean objects and the features (eigenfaces) are presented and discussed.

Authors test the possibilities of using the set of eigenvectors and coefficient values (computed from PCA) for a security verification. As an example of authorization code the sets of coefficient values for the faces and hands are presented. Each code describes individual features and can be decoded and compared with the original data of user to obtain access to restricted area or information.

**REFERENCES**

[1] R. Sanchez-Reillo, C. Sanchez-Avila and A. Gonzales-Marcos. Biometric Identification Through Hand Geometry Measurements. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **22** (10):1168–1171, 2000.

[2] F. Prokoski. History, Current Status, and Future of Infrared Identification. In: *Proc. of the IEEE Workshop on Computer Vision behind the Visible Spectrum: Methods and Applications (CVBVS 2000)*, , 2000, 5 – 14.

[1]This work was supported under research grant no: N518 496039 from the Polish Ministry of Science.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON PRICE EQUILIBRIUM OF STOCHASTIC MARKET

KĀRLIS ŠADURSKIS

*Riga Technical University*

Meža iela 1/4, Rīga, LV-1048, Latvia

E-mail: karlis.sadurskis@rtu.lv

The paper deals with statistical model of an adaptive Samuel-Marshall type single component market [1; 3]. Our model supposes that a manufacturer has a monopoly there and he would like to stabilize the price of a product unit into a small neighborhood of the level $\bar{p}$. Let us remind, that in any classical single market model a price equilibrium $p(t) \equiv \bar{p}$ can be achieved by the equality of demand $D(\bar{p})$ to supply $S(\bar{p})$ [1; 3]. To control a price at the time moment $t$ the manufacturer can use a supplied quantity $S_t$, but to enter the market he needs some time $h$.Therefore manufacturer is entering the market at the moments of time $t$ and thus he has a delayed reaction because he is guided by the price at the moment of time $t-h$. As a result the supply $S_t$ depends on the price $p(t-h)$. The demand $D_t$ at the time $t$ instantaneously affects on the price value , i.e., $D_t := D(p(t))$. As in the classical Samuelson model [3] we will suppose equilibrium to be reached due to an adaptive price dynamical property: the price movement $(\Delta p)(t) := p(t+\triangle)-p(t)$ is proportional to difference $D_t - S_t$ multiplied by time increment $\triangle$. This assumption permits to analyze price equilibrium stability writing out a market mathematical model in a form of the first order ordinary differential equation with delay

$$\frac{dp(t)}{dt} = D(p(t)) - S(p(t-h)$$

where demand and supply linearly dependent on price. Applying the second Lyapunov method for delayed stochastic differential equation [4] our paper analyzes linearized equation for deviations $x(t) = p(t) - \bar{p}$

$$dx(t) = b(cx(t) - x(t-h))dt + \sigma x(t)dw(t). \tag{1}$$

where $w(t)$ is standard Wiener process. Parameter $\sigma$ (called by *volatility*) allows to take into account value at risk connected with this market model [5].

**REFERENCES**

[1] A.Marshall. *Principles of Economics*. Macmillan Press Ltd., NY, 1979.

[2] R.Merton. *Contnuous-Time Finance*. Blackwell, Cambridge, UK, 1990.

[3] P.A.Samuelson. *Foundations of Economic Analysis*. Atheneum, NY, 1974.

[4] J. F. Carkov. *Random Perturbations of Functional Differential Equations*. Zinatne, Riga, 1989, (in Russian).

[5] W.Sharpe. Capital Asset Prices: a Theory of Market Equilibrium under Conditions of Risk. *Journal of Finance*, September: 425–442,1964.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# THE WEIGHTED FINITE-DIFFERENCE SCHEME FOR TWO-DIMENSIONAL PARABOLIC EQUATION WITH FOUR NONLOCAL INTEGRAL CONDITIONS

SVAJŪNAS SAJAVIČIUS

*Vilnius University*

Naugarduko 24, LT-03225, Vilnius, Lithuania

E-mail: svajunas.sajavicius@mif.vu.lt

*Mykolas Romeris University*

Ateities 20, LT-08303, Vilnius, Lithuania

E-mail: svajunas@mruni.eu

We construct and analyse the weighted finite-difference scheme for two-dimensional parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f(x,y,t), \quad 0 < x < 1, \quad 0 < y < 1, \quad 0 < t \leqslant T,$$

subject to nonlocal integral conditions

$$u(0,y,t) = \gamma_1 \int_0^1 \alpha_1(x) u(x,y,t) \mathrm{d}x + \mu_1(y,t),$$

$$u(1,y,t) = \gamma_2 \int_0^1 \alpha_2(x) u(x,y,t) \mathrm{d}x + \mu_2(y,t), \quad 0 < y < 1, \quad 0 < t \leqslant T,$$

$$u(x,0,t) = \gamma_3 \int_0^1 \alpha_3(y) u(x,y,t) \mathrm{d}y + \mu_3(x,t),$$

$$u(x,1,t) = \gamma_4 \int_0^1 \alpha_4(y) u(x,y,t) \mathrm{d}y + \mu_4(x,t), \quad 0 < x < 1, \quad 0 < t \leqslant T,$$

and initial condition

$$u(x,y,0) = \varphi(x,y), \quad 0 \leqslant x \leqslant 1, \quad 0 \leqslant y \leqslant 1,$$

where $f(x,y,t)$, $\mu_1(y,t)$, $\mu_2(y,t)$, $\mu_3(x,t)$, $\mu_4(x,t)$, $\alpha_1(x)$, $\alpha_2(x)$, $\alpha_3(y)$, $\alpha_4(y)$, $\varphi(x,y)$ are given functions, $\gamma_1$, $\gamma_2$, $\gamma_3$, $\gamma_4$ are given parameters, and function $u(x,y,t)$ is unknown.

The main attention is paid to the stability of scheme. We use the well-known stability analysis technique which was used in order to analyse the finite-difference schemes for differential problems with other types of nonlocal conditions (see, e.g., [1]). We demonstrate the efficiency of the considered finite-difference scheme by solving several test problems with different types of weight functions $\alpha(x)$ and $\beta(x)$ and calculating the maximum norm of computational errors. All numerical experiments were performed using the technologies of grid computing.

**REFERENCES**

[1] M. Sapagovas, G. Kairytė, O. Štikonienė and A. Štikonas. Alternating direction method for a two-dimensional parabolic equation with a nonlocal boundary condition. *Math. Model. Anal.*, **12** (1):131–142, 2007.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# DIFFERENCE METHODS FOR DIFFERENTIAL FUNCTIONAL PARABOLIC EQUATIONS AND ESTIMATES OF SOLUTIONS

LUCJAN SAPA

*Faculty of Applied Mathematics,*
*AGH University of Science and Technology*

Al. Mickiewicza 30, 30-059 Kraków, Poland

E-mail: lusapa@mat.agh.edu.pl

KAROLINA KROPIELNICKA

*Institute of Mathematics,*
*University of Gdańsk*

Ul. Wita Stwosza 57, 80-952 Gdańsk, Poland

karolina.kropielnicka@math.univ.gda.pl

Assume that, $T > 0$, $X = (X_1, ..., X_n)$, $\tau_0 \geq 0$, $\tau = (\tau_1, ..., \tau_n)$, where $X_i > 0$, $\tau_i \geq 0$ for $i = 1, ..., n$. Define $E = [0, T] \times (-X, X) \subset \mathbf{R}^{1+n}$, $E_0 = [-\tau_0, 0] \times [-X - \tau, X + \tau] \subset \mathbf{R}^{1+n}$, $\partial_0 E = [0, T] \times ([-X - \tau, X + \tau] \setminus (-X, X)) \subset \mathbf{R}^{1+n}$, $\Omega = E \cup E_0 \cup \partial_0 E$. Define also $\Delta = E \times C(\Omega, \mathbf{R}) \times \mathbf{R}^n \times M_{n\times n}$, where $M_{n\times n}$ is the set of all $n \times n$ symmetric real matrices.

Let $f : \Delta \to \mathbf{R}$ and $\varphi : E_0 \cup \partial_0 E \to \mathbf{R}$ be given functions. Consider a nonlinear second-order partial differential functional equation of parabolic type of the form

$$\partial_t z(t, x) = f(t, x, z, \partial_x z(t, x), \partial_{xx} z(t, x)) \tag{1}$$

with the *initial condition* and the *boundary condition of the Dirichlet type*

$$z(t, x) = \varphi(t, x) \text{ on } E_0 \cup \partial_0 E, \tag{2}$$

where $\partial_x z = (\partial_{x_1} z, ..., \partial_{x_n} z)$, $\partial_{xx} z = \left[\partial_{x_i x_j} z\right]_{i,j=1}^{n}$. Our aim is to give explicit and implicit convergent and stable difference methods for the initial boundary problem (1), (2).

The functional dependence is of the Volterra type (e.g., delays or Volterra type integrals). The equation may be nonlinear with respect to second derivatives. The nonlinear estimate of the generalized Perron type for $f$ introduced in [2] is assumed. Under this generalization, equation (1) includes, as special cases, a quasi-linear equation and a strongly nonlinear equation with a quasi-linear term.

If $f$ is, e.g., a polynomial with respect to the functional variable, then the global generalized Perron type estimate is not fulfilled. But we give two groups of theorems on the estimates of solutions for the differential functional problem (1), (2) and a family of associated difference functional schemes. By these theorems, the numerical methods may be treated in the subspace $C(\Omega, R) \subset C(\Omega, \mathbf{R})$, $R \subset \mathbf{R}$ is some interval, where we consider the local generalized Perron type condition ([1]). Hence equation (1) covers equations with the polynomial right-hand sides, e.g., the Fisher equation, the porous media equation, the Newell-Whitehead equation, the Zeldovich equation, the KPP equation, the Nagumo equation, the Huxley equation and others.

**REFERENCES**

[1] K. Kropielnicka, L. Sapa. Estimate of solutions for differential and difference functional equations with applications to difference methods. *Appl. Math. Comput.*, **217** (13): 6206–6218, 2011.

[2] L. Sapa. A finite difference method for quasi-linear and nonlinear differential functional parabolic equations with Dirichlet's contition. *Ann. Polon. Math.*, **93** (2): 113–133, 2008.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# THE EIGENVALUE PROBLEM FOR DIFFERENTIAL AND DIFFERENCE OPERATORS WITH NONLOCAL CONDITIONS: THEORY AND APPLICATIONS

MIFODIJUS SAPAGOVAS

*Vilnius University, Institute of Mathematics and Informatics*

Akademijos 4, LT-08663 Vilnius, Lithuania

E-mail: mifodijus.sapagovas@mii.vu.lt

During the last several decades the differential operators with nonlocal conditions were an important research topic of mathematics, in particular, of numerical analysis. Quite a new area, related with the problems of this type, is an eigenvalue problem for differential (or difference) operators with nonlocal conditions.

Very simple examples show, how the eigenvalue problem with nonlocal conditions reduces to a more complicated, more rich in content, and more interesting problem. One of the simple problems is [1]

$$\frac{d^2u}{dx^2} + \lambda u = 0, \quad 0 < x < 1, \tag{1}$$

$$u(0) = 0, u(1) = \gamma u(\xi), \quad 0 < \xi < 1. \tag{2}$$

In this plenary lecture, a typical problem to be considered is the eigenvalue problem for equation (1) as well as a difference analogue of this equation with the nonlocal conditions

$$u(0) = \gamma_1 \int_0^1 \alpha(x)u(x)dx, \quad u(1) = \gamma_2 \int_0^1 \beta(x)u(x)dx. \tag{3}$$

The main point is to determine the conditions under which there exist the negative eigenvalues or complex eigenvalues with a negative real component. The eigenvalues of this type have a negative influence on the stability of difference schemes for a parabolic equation as well as on the convergence of iterative methods for elliptic equations.

The results of numerical experiments are presented as well.

**REFERENCES**

[1] M.P. Sapagovas and A.D. Štikonas. On the structure of the spectrum of a differential operator with a nonlocal condition. *Diff. Equ.*, **41** (7):961–969, 2005.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# NUMERICAL SOLUTION OF NON-LINEAR LAGRANGIAN EQUATION FOR INERTIAL PARTICLES IN TURBULENT FLOWS

MIHAILS ŠČEPANSKIS[1], ANDRIS JAKOVIČS[1], EGBERT BAAKE[2] and BERNARD NACKE[2]

[1] *University of Latvia, Faculty of Physics and Mathematics, Department of Physics, Laboratory for Mathematical Modelling of Environmental and Technological Processes*

Zeļļu iela 8, LV-1002, Rīga, Latvia

[2] *Leibniz University of Hanover, Institute of Electrotechnology*

Wilhelm-Busch-Str. 4, D-30167, Hannover, Germany

E-mail: `mihails.scepanskis@lu.lv`

The solid particle transport problem in recirculated turbulent electromagnetically (EM) induced metal melt flows of industrial electroheat equipment – induction crucible furnaces (ICF) and channel induction furnaces – is considered in the present paper. The flow pattern can be described with one-way coupled equations of EM and hydrodynamics (HD) and is researched in [1]. The deposition of the impurities from molten dirty secondary metals as well as the problem of the homogenization of the alloying particles are associated with the particle transfer problem, which is considered in the paper.

The EM problem is solved within finite element method, and the HD equations, which takes into account the calculated EM source, are solved within finite volume method using Large Eddy Simulation approach. The results of the HD problem (velocity and acceleration fields) are used for the calculation of the particle trajectories within the bounds of one-way coupling. The following Lagrangian equation for particle motion is obtained and analyzed in the paper:

$$\left(1+\frac{C_A}{2}\frac{\rho_f}{\rho_p}\right)\cdot\frac{d\mathbf{u_p}}{dt}=C_D\cdot\mathbf{U}+\left(1-\frac{\rho_f}{\rho_p}\right)\cdot\mathbf{g}-\frac{3}{4}\frac{1}{\rho_p}\mathbf{f_{em}}+\frac{\rho_f}{\rho_p}C_L\xi+\left(1+\frac{C_A}{2}\frac{\rho_f}{\rho_p}\right)\cdot\frac{D\mathbf{u_f}}{Dt},\quad(1)$$

where $\mathbf{U}=\mathbf{u_f}-\mathbf{u_p}$, $\mathbf{u_f}$ and $\mathbf{u_p}$ are liquid and particle velocities respectively, $\rho_f$ and $\rho_p$ are liquid and particle density respectively, $\mathbf{g}$ is free fall acceleration; $\mathbf{f_{em}}=\frac{1}{2}[\mathbf{j}\times\mathbf{B}^*]$ is the avaraged Lorenz force density, $\mathbf{j}$ is current density, $\mathbf{B}^*$ is complex conjugated magnetic field induction, $\xi=[\mathbf{U}\times[\nabla\times\mathbf{U}]]$; $C_A(dU/dt,U)$, $C_D(U)$ and $C_L(U)$ are acceleration, drag and lift force coefficients respectively.

The HD step of time is splited to various Lagrangian steps of time (LST). The non-linear factors in equation (1) are approximated with the values, which correspond to the previous LST (Pikar's method). Subsequently the implicite scheme is used to solve the equation. The convergence of this numerical scheme for concerned HD case is analized numerically (the results of the simulations for different LST are compared).

The algorithm described above is coded by means of OpenFOAM. Therefore the numerical approach for the simulation of solid inertial particles in the turbulent EM induced flows is developed and investigated.

**REFERENCES**

[1] M. Kirpo. *Modeling of Turbulence Properties and Particle Transport in recirculated Flows (PhD thesis)*. University of Latvia, Faculty of Physics ans Mathematics, Department of Physics, Riga, 2008.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# STRONG SUMMABILITY IN CERTAIN FAMILIES OF SUMMABILITY METHODS FOR FUNCTIONS

ANNA ŠELETSKI

*Tallinn University*

Narva road 25, Tallinn, Estonia

E-mail: annatar@hot.ee

Some families of functional summability methods are discussed. The results introduced here extend the results published in [3] and [4].

1. Let us denote by $X$ the set of the functions $x = x(u)$ defined for $u \geq 0$, bounded and measurable by Lebesgue on every finite interval $[0, u_0]$. Suppose that $A$ is a transformation of functions $x = x(u)$ (or, in particular, of sequences $x = (x_n)$) into functions $Ax = y = y(u) \in X$. If the limit $\lim_{u\to\infty} y(u) = s$ exists then we say that $x = x(u)$ is convergent to $s$ with respect to the summability method $A$, and write $x(u) \to s(A)$.

2. We consider a Riesz-type family $\{A_\alpha\}$ of summability methods $A_\alpha$, where $\alpha > \alpha_0$ and $\alpha_0$ is some fixed number, and which transform functions $x = x(u)$ into functions $A_\alpha x = y_\alpha(u)$. This family is defined with the help of relation $A_\beta = C_{\gamma,\beta} \circ A_\gamma \quad (\beta > \gamma > \alpha_0)$, where $C_{\gamma,\beta}$ is certain integral transformation (see e.g. [3]). For example, the Riesz methods $(R, \alpha)$, certain generalized Nörlund methods $(N, p_\alpha(u), q(u))$ and Borel methods $(B, \alpha)$ form Riesz-type families.

3. In this talk the strong summability methods $[A_{\alpha+1}]_k$ in a Riesz-type family $\{A_\alpha\}$ are defined with the help of some positive function $k = k(u) \in X$. These strong summability methods are compared for different values of parameter $\alpha$ by their summability fields and by speeds of convergence. Also, the methods $[A_{\alpha+1}]_k$ are compared with ordinary summability methods $A_\alpha$ and $A_{\alpha+1}$. The proved theorems generalize different results already published for matrix methods (see [1] and [2]), and are applied, in particular, to Riesz methods, generalized integral Nörlund methods and Borel methods. For example, the following implications are proved for any $\beta > \delta > \gamma > \alpha_0$ :

$$x(u) \to s[A_{\gamma+1}]_k \Longrightarrow x(u) \to s(A_{\gamma+1}),$$

$$x(u) \to s[A_{\gamma+1}]_k \Longrightarrow x(u) \to s[A_{\beta+1}]_k,$$

$$x(u) = O([A_{\gamma+1}]_k), \quad x(u) \to s[A_{\beta+1}]_k \Longrightarrow x(u) \to s[A_{\delta+1}]_k.$$

Comparative estimates for speeds of $[A_{\alpha+1}]_k$-convergence are also given in the talk.

**REFERENCES**

[1] Soomer, V. and Tali, A. On strong summability of sequences, *Acta Comment. Univ. Tartu. Math.*, 2007, **11**, 57–68.

[2] Stadtmüller, U. and Tali, A. Strong summability in certain families of summability methods, *Acta Sci. Math. (Szeged)*, 2004, **70**, 639–657.

[3] Šeletski, A. and Tali, A. Comparison of speeds of convergence in Riesz-type families of summability methods, *Proc. Estonian Acad. Sci.*, **57** (2), 2008, 70–34.

[4] Šeletski, A. and Tali, A. Comparison of speeds of convergence in Riesz-type families of summability methods. II, *Math. Model. Anal.*, 2010, **15**, 103–112 .

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# THE EXISTENCE OF SOLUTION FOR SOME NONLOCAL PROBLEM

NATALIJA SERGEJEVA

*Latvia University of Agriculture*

Lielā iela 2, Jelgava, LV-3001, Latvia

E-mail: natalijasergejeva@inbox.lv

The existence result is established for the nonlocal boundary problem

$$x'' = -\mu x^+ + \lambda x^- + h(t, x, x') \tag{1}$$

$$x(0) = 0,\ x(1) + \int_0^1 x(s)ds = 0, \tag{2}$$

provided that $h(t, x, x')$ is bounded nonlinearity and employing properties of the Fučík spectrum for the related Fučík type nonlocal problem

$$x'' = -\mu x^+ + \lambda x^- \tag{3}$$

with conditions (2).

**REFERENCES**

[1] A. Gritsans, F. Sadyrbaev and N. Sergejeva. Two-parameter nonlinear eigenvalue problems. In: *Proceedings of the International Conference on Boundary Value Problems, Santiago de Compostela, Spain, 2008*, Mathematical Models in Engineering, Biology, and Medicine, A. Cabada, E. Liz and J.J. Nieto (Eds.), American Institute of Physics Conference Proceedings, Vol. 1124, 2009, 185 – 194.

[2] N. Sergejeva. On Some Problems with Nonlocal Integral Condition. *Math. Model. Anal.*, **15** (1):113–126, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# FINITE DIFFERENCE SCHEME ON ADAPTED LOCALLY-UNIFORM GRIDS FOR SINGULARLY PERTURBED PARABOLIC EQUATIONS AND ITS STABILITY TO PERTURBATIONS IN THE DATA[1]

GRIGORY SHISHKIN

*Institute of Mathematics and Mechanics, Russian Academy of Sciences*

S. Kovalevskaya Str., 16, GSP-384, Ekaterinburg, Russia

E-mail: shishkin@imm.uran.ru

Finite difference schemes on *uniform grids* developed for regular problems have a series of advantages. So, they permit to construct schemes of *high-order accuracy*, to use *effective methods for solving* discrete problems (see, e.g., [1; 2]). But the use of uniform grids for solving singularly perturbed problems (with a small parameter $\varepsilon$, $\varepsilon \in (0, 1]$, multiplying the high-order derivative in the equation) causes difficulties. For example, a classical scheme on the uniform grid, under the condition of its convergence (for $N \gg \varepsilon^{-1}$ where $N$ defines the number of nodes in the mesh in $x$), *is not $\varepsilon$-uniformly well-conditioned*. From this, it follows that in order to solve the problem with a prescribed accuracy it is necessary: *to increase* $N$ unboundedly ($N \gg \varepsilon^{-3/2}$ for stability of the scheme) and to use a computer with unboundedly *growing number of digits* in the computer word.

In the present talk, conditioning of a difference scheme is discussed that was developed in [3; 4] for a Dirichlet problem to a singularly perturbed parabolic convection-diffusion equation. For this problem, monotone classical (of the first accuracy order) approximations of differential equations on adapted *locally-uniform grids* are used. These grids are *uniform on subdomains* where the computed solution is corrected. Boundaries of such subdomains are determined by a *majorant* of the singular component of the discrete solution. On the basis of such approach, a difference scheme is constructed that converges *almost $\varepsilon$-uniformly*, i.e., the convergence rate of the scheme *weakly depend* on the value of the parameter $\varepsilon$ (see, e.g., [4]); for *not too small* values of the parameter, the scheme converges with the *first accuracy order*. Unlike [3; 4], in the present research for studying conditioning of the difference scheme, more deep analysis of justification to the construction of the difference scheme is realized and more accurate *a priori* estimates are derived. It is proved that the constructed scheme on *locally-uniform* grids is *$\varepsilon$-uniformly well-conditioned*, i.e., the *conditioning number* of the scheme is *$\varepsilon$-uniformly bounded*, from this follows *$\varepsilon$-uniform stability* of the discrete solution to perturbations in the data of the difference scheme (arising in a computational process).

**REFERENCES**

[1] G. Marchuk and V. Shaidurov. *Difference Methods and Their Interpolations*. Springer–Verlag, New York, 1983.

[2] A. Samarskii. *The Theory of Difference Schemes*. Marcel Dekker, New York, 2001.

[3] G. Shishkin. A finite difference scheme on a priori adapted meshes for a singularly perturbed parabolic convection-diffusion equation. *Numer. Math. Theor. Meth. Appl.*, **1** (2):214–234, 2008.

[4] G. Shishkin and L. Shishkina. *Difference Methods for Singular Perturbation Problems*. Chapman and Hall/CRC, London New York, 2009.

[1] This research was supported by the Russian Foundation for Basic Research under grant 10-01-00726.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# APPROXIMATION OF SOLUTIONS AND DERIVATIVES TO $\mathcal{E}$-UNIFORMLY CONVERGENT DIFFERENCE SCHEMES, USING UNIFORM GRIDS, FOR SINGULARLY PERTURBED PROBLEMS[1]

LIDIA SHISHKINA and GRIGORY SHISHKIN

*Institute of Mathematics and Mechanics, Russian Academy of Sciences*

S. Kovalevskaya Str., 16, GSP-384, Ekaterinburg, Russia

E-mail: lida@convex.ru

Approximation of solutions and derivatives is studied for a Dirichlet problem to a singularly perturbed ordinary differential reaction-diffusion equation with a small parameter $\varepsilon$ ($\varepsilon \in (0,1]$) multiplying the high-order derivative in the equation. For this problem, an approach — based on a *decomposition of the discrete solution* into the regular and singular components which are solutions of *discrete subproblems* using *uniform grids* — is applied; a description of this method can be find in [1; 2; 3]). Using an *asymptotic construction technique*, a *difference scheme* of the *solution decomposition method* is constructed that converges $\varepsilon$-uniformly in the maximum norm at the rate $\mathcal{O}\left(N^{-2}\ln^2 N\right)$, where $N+1$ is the number of nodes in the grids used; for fixed values of the parameter $\varepsilon$, the scheme converges at the rate $\mathcal{O}\left(N^{-2}\right)$. This scheme allows us to approximate the *regular and singular components of the solution and their derivatives* up to the second order. A modified scheme of the solution decomposition method is constructed for which the regular component of the solution and its discrete derivatives converge $\varepsilon$-uniformly in the maximum norm at the rate $\mathcal{O}\left(N^{-2}\right)$ for sufficiently smooth values of $\varepsilon$ (for $\varepsilon = o(\ln^{-1} N)$). For the scheme of the solution decomposition method under the condition $\varepsilon = o(\ln^{-1} N)$, the convergence rate of the *regular and singular components* of the discrete solution and *their grid derivatives* up to the second order *are better* than the convergence rate of the discrete solution and its grid derivatives for the *scheme on piecewise-uniform grids*.

**REFERENCES**

[1] G. Shishkin and L. Shishkina. Improved difference scheme of the solution decomposition method for a singularly perturbed reaction-diffusion equation. *Proceedings of the Steklov Institute of Mathematics*, **272** (1):S197–S214, 2011.

[2] G. Shishkin. Difference scheme of the solution decomposition method for a singularly perturbed parabolic reaction-diffusion equation. *Russian Journal of Numerical Analysis and Mathematical Modelling*, **25** (3):261–278, 2010.

[3] G. Shishkin and L. Shishkina. A Richardson Scheme of the Decomposition Method for Solving Singularly Perturbed Parabolic Reaction-Diffusion Equation. *Comp. Math. Math. Phys.*, **50** (12):2003–2022, 2010.

[1] This research was supported by the Russian Foundation for Basic Research under grant 10-01-00726.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# FORECASTING LATVIAN RESIDENTIAL PROPERTY PRICES USING STRUCTURAL TIME SERIES MODELS

NADEZHDA SINENKO and ARTJOMS JEFIMOVS

*Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: sinenko@latnet.lv, artjomsj@gmail.com

Presence of a turning point at the end of a sample introduces degree of uncertainty in econometric forecasting. Deterministic trend-based forecasting strategies are not relevant in this case. Our article investigates possibilities of using unobserved components model with time varying parameters to improve forecasts of a time series in the vicinity of turning point. A structural time series model is set in terms of components having a direct economical interpretation.Young [4] considered additive structural model in a form

$$y_t = T_t + S_t + \epsilon_t, \quad t = 1, \ldots, T, \tag{1}$$

where cyclical movements are incorporated into trend component $T_t$, $S_t$ describes seasonal variation and irregular component $\epsilon_t$ is assumed to be white noise with zero mean and variance $\sigma_\epsilon^2$. The new framework elaborated for structural models by Harvey (see e.g. [2], [3]) made structural models more flexible by letting the level and slope parameters of trend to change over time:

$$T_t = T_{t-1} + D_{t-1} + \zeta_t, \quad \zeta_t \sim NID(0, \sigma_\zeta^2), \tag{2}$$

$$D_t = D_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2), \tag{3}$$

where $D_t$ is the slope and the white noise disturbances $\zeta$ and $\eta$ are independent of each other. This way of modelling time trend allows for time-varying parameters and incorporates possibilities of random walk and linear trend as limiting cases. Estimation and forecasting of the model is produced within a unified statistical framework, using Kalman filter. Garcia-Ferrer in [1] proposed to improve forecasts, obtained in integrating random walk specification, by fitting to extracted trend derivative $D_t$ an autoregressive model. We compare this approach with estimation of autoregressive parameters of trend derivative $D_t$ together with other components by Kalman filter, substituting random walk specification in equation (3) by AR(p) specification.

The purpose of the article is to investigate the forecasting performance of both methods applied to Latvian residential property prices and compare results with traditional ARIMA benchmark.

**REFERENCES**

[1] A.Garcia-Ferrer, R.A Queralt. Can univariate models forecast turning points in seasonal economic time series. *International Journal of Forecastingl*, **14** pp. 433-446, 1998.

[2] A.C. Harvey. *Forecasting, Structural Time series Models and the Kalman Filter*. Cambridge University Press, Cambridge, 1989.

[3] A.C. Harvey, A. Jaeger. Detrending, stylized facts and the business cycle. *Journal of applied Econometrics*, **8** pp. 31-47, 1993.

[4] P.C. Young. Time variable parameter and trend estimation in non-stationary economic time series. *Journal of Forecasting*, **13** No. 2, pp. 179-210, 1994.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# COMPARISON OF DIFFERENT NUMERICAL SOLUTIONS OF GROUNDWATER INFLOW INTO A TUNNEL

ILONA SKARYDOVA and MILAN HOKR

*Institute of New Technologies and Applied Informatics, Technical University of Liberec*

Studentska 2, 461 17 Liberec 1, the Czech Republic

E-mail: ilona.skarydova@tul.cz, milan.hokr@tul.cz

Understanding coupled thermo-hydro-mechano-chemical (THMC) processes in rock is a fundamental point in choosing the best site for geological disposal of nuclear waste. It is important to study the behaviour of numerical methods on problems with real-world data. We solve groundwater and surface water flow in the fractured granite, using digital terrain model and field measured data from water supply tunnel in Bedrichov in the northeast of the Czech Republic, a 30 years old analogy for the repository tunnels with similar geological conditions.

We use own simulation software Flow123D, [3] which is based on mixed-hybrid formulation of the finite element method and one of its features is coupling 3D, 2D and 1D domains in one problem, [2]. In our models, 3D elements represent rock equivalent continuum (groundwater flow), 2D elements surface water flow (with the rainfall as the source) and 1D elements flow in the rivers. The problem is solved as potential flow (Darcy law and continuity equation, [1]). Model of potential flow is an empirical replace of physical equations of surface water flow.

We study several numerical issues related both to the method and the data. Results of the 3D model are compared with results of 2D model representing vertical cross-section, to observe effect of mesh quality on the accuracy. Effect of massif inhomogeneity (layers of different hydraulic conductivity and vertical conductive planes fractures, versus fully homogeneous model) is compared in terms of quality of measured data fitting as well as numerical behaviour. Performance of algebraic solvers is evaluated in relation to the problem geometric features (km problem scale to metres tunnel scale), mesh size and material parameters.

**REFERENCES**

[1] O.D.L. Strack. *Groundwater Mechanics*. Prentice Hall, 1989.

[2] J. Maryska, O. Severyn, M. Tauchman and D. Tondr. Modelling of processes in fractured rock using FEM/FVM on multidimensional domains. *Journal of Computational and Applied Mathematics*, **215** (2):495–502, 2008.

[3] O. Severyn, M. Hokr, J. Kralovcova, J. Kopal and M. Tauchman. *Flow123D: Numerical simulation software for flow and solute transport problems in combination of fracture network and continuum.* Technical report, TU Liberec. 2008

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# INVESTIGATION STURM–LIOUVILLE PROBLEMS WITH INTEGRAL BOUNDARY CONDITION

AGNE SKUČAITĖ

*Institute of Informatics and Mathematics, Vilnius University*

Akademijos 4, LT-08663 Vilnius, Lithuania

E-mail: agne.skucaite@mii.vu.lt

We investigate Sturm–Liouville problem with one classical boundary condition

$$-u'' = \lambda u, \quad t \in (0,1), \qquad u(0) = 0, \tag{1}$$

and another NBC:

$$u(1) = \gamma \int_0^{\xi} u(t)\mathrm{d}x \text{ or } u(1) = \gamma \int_{\xi}^{1} u(t)\mathrm{d}x \tag{2}$$

with parameters $\gamma \in \mathbb{R}$ and $\xi \in (0,1)$.

On the interval $[0,1]$, we introduce uniform grids $\bar{\omega}^h = \{x_j = jh, j = 0,1,...,n; nh = 1\}$, $\omega^h = \bar{\omega}^h \setminus \{x_0, x_n\}$. Also, we make the assumption, that $\xi$ coincide with any grid-point, i.e. $\xi = mh = m/n$, here $0 < m < n$. We approximate differential problems (1)–(2) by the following discrete problem:

$$\frac{y_{j-1} - 2y_j + y_{j+1}}{h^2} + \lambda y_j = 0, \quad j \in \omega^h, \qquad y_0 = 0, \tag{3}$$

and one of these NBC (the trapezoidal rule):

$$y_n = \gamma h\Big(\sum_{i=1}^{m-1} y_i + \frac{y_m}{2}\Big); \; y_n = \gamma h\Big(\sum_{i=m+1}^{n-1} y_i + \frac{y_m + y_n}{2}\Big), \tag{4}$$

or (the Simpson rule)

$$y_n = \frac{\gamma h}{3}\Big(y_{2m} + y_{2n} + 4\sum_{i=m+1}^{n} y_{2i-1} + 2\sum_{i=m+1}^{n-1} y_{2i}\Big); \; y_n = \frac{\gamma h}{3}\Big(y_0 + y_{2m} + 4\sum_{i=1}^{m} y_{2i-1} + 2\sum_{i=1}^{m-1} y_{2i}\Big). \tag{5}$$

We analyze how the spectrum of these problems (differential and two discrete) depends on the NBC parameters $\gamma$ and $\xi$.

**REFERENCES**

[1] A. Štikonas and O. Štikonienė. Characteristic Functions for Sturm-Liouville Problems with Nonlocal Boundary Conditions. *Mathematical Modelling and Analysis* , **14** (2):229–246, 2009.

[2] A. Skučaitė, K. Skučaitė-Bingelė, S. Pečiulytė and A. Štikonas. Investigation of the Spectrum for the Sturm-Liouville Problem with one Integral Boundary Condition. *Nonlinear Analysis: Modelling and Control*, **15** (4):501–512, 2010.

[3] M. Sapagovas. *Diferencialinių lygčių kraštiniai uždaviniai su nelokaliosiomis sąlygomis*. Mokslo aidai, Vilnius, 2007. (in Lithuanian)

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# INVESTIGATION OF COMPLEX EIGENVALUES FOR STATIONARY PROBLEMS WITH TWO-POINT NONLOCAL BOUNDARY CONDITION

KRISTINA SKUČAITĖ-BINGELĖ

*Institute of Informatics and Mathematics, Vilnius University*

Akademijos 4, LT-08663 Vilnius, Lithuania

*Vytautas Magnus University*

Vileikos g. 8, LT - 44404 Kaunas, Lithuania

E-mail: Kristina.Skucaite-Bingele@fc.vdu.lt

Let us investigate the Sturm–Liouville problem

$$-u'' = \lambda u, \quad t \in (0,1), \tag{1}$$

with one classical (the first or the second type) boundary condition: $u(0) = 0$ or $u'(0) = 0$; and another two-point boundary condition ($0 \leqslant \xi \leqslant 1$):

$$u'(1) = \gamma u(\xi), \qquad u'(1) = \gamma u'(\xi), \qquad u(1) = \gamma u'(\xi), \qquad u(1) = \gamma u(\xi). \tag{2}$$

with parameter $\gamma \in \mathbb{R}$, $\xi \in (0,1)$.

On the interval $[0,1]$, we introduce uniform grids $\bar{\omega}^h = \{x_j = jh, j = 0,1,...,n; nh = 1\}$, $\omega^h = \bar{\omega}^h \setminus \{x_0, x_n\}$. Also, we make the assumption, that $\xi$ coincide with any grid-point, i.e. $\xi = mh = m/n$, here $0 < m < n$. We approximate differential problems (1)–(2) by the following discrete problem:

$$\frac{y_{j-1} - 2y_j + y_{j+1}}{h^2} + \lambda y_j = 0, \quad j \in \omega^h, \tag{3}$$

and with one BC:

$$y_0 = 0 \text{ or } y_1 = y_0; \tag{4}$$

$$y_n = \gamma y_m \text{ or } \quad y_n = \gamma \frac{y_{m+1} - y_{m-1}}{2h}. \tag{5}$$

We analyze such Sturm–Liouville problems and investigate how the spectrum in the complex plane of these problems depends on the nonlocal boundary conditions parameters $\gamma$ and $\xi$.

**REFERENCES**

[1] K. Skučaitė, S. Pečiulytė and A. Štikonas. Investigation of Complex Eigenvalues for Stationary Problems with two-point Nonlocal Boundary Conditions. *Mathematics and Mathematical Modelling–2009*, KTU, Kaunas, Lietuva, 2009.

[2] A. Skučaitė, K. Skučaitė-Bingelė, S. Pečiulytė and A. Štikonas. Investigation of the Spectrum for the Sturm-Liouville Problem with one Integral Boundary Condition. *Nonlinear Analysis: Modelling and Control*, **15** (4):501–512, 2010.

[3] A. Štikonas and O. Štikoniene. Characteristic Functions for Sturm-Liouville Problems with Nonlocal Boundary Conditions. *Mathematical Modelling and Analysis* , **14** (2):229–246, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON SOME THIRD ORDER NONLINEAR BOUNDARY VALUE PROBLEMS WITH NONLOCAL BOUNDARY CONDITIONS[1]

SERGEY SMIRNOV

*Daugavpils University*

Parādes iela 1, Daugavpils LV-5400, Latvia

E-mail: srgsm@inbox.lv

The third order Emden-Fowler type equation

$$x''' = -|x|^p \operatorname{sign} x \tag{1}$$

together with the nonlocal boundary conditions

$$x(0) = x'(0) = 0, \quad \int_0^1 x(s)ds = 0 \tag{2}$$

is considered. We treat the number of solutions to the problem (1), (2). Examples are included to illustrate the results.

**REFERENCES**

[1] E. Rovderova. On the number of solutions of a third-order boundary-value problem. *Trans. Amer. Math. Soc.*, **347** : 3079–3092, 1995.

[2] C. de Coster and M. Gaudenzi. On the existence of infinitely many solutions for superlinear n-th order boundary value problems. *Nonlinear World*, **4** : 505–524, 1997.

[3] J.R. Graef, B. Yang. Positive solutions of a third order nonlocal boundary value problem. *Discrete Contin. Dyn. Syst.*, **1** : 89–97, 2008.

[4] P. Bailey, L. Shampine, P. Waltman. *Nonlinear two point boundary value problems*. Academic Press, New York, 1968.

[5] S. Bernfeld and V. Lakshmikantham. *An Introduction to Nonlinear Boundary Value Problems*. Academic Press, New York, 1974.

[6] R.P. Agarwal. *Boundary Value Problems for Higher Order Equations*. World Scientific Publishing, 1986.

[7] I. Kiguradze, T.A. Chanturia. *Asymptotic Properties of Solutions of Nonautonomous Ordinary Differential Equations*. Springer, 1992.

[1]ESF projekts "Atbalsts Daugavpils Universitātes doktora studiju īstenošanai" Vienošanās Nr. 2009/0140/1DP/1.1.2.1.2/09/IPIA/VIAA/015

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# COUPLED EM AND HD NUMERICAL CALCULATION OF FREE SURFACE DYNAMICS OF MELT IN ICF

S. SPITANS[1], A. JAKOVICS[1], B. NACKE[2] and E. BAAKE[2]

[1] *Laboratory for Mathematical Modelling of Environmental and Technological Processes, UL*

Zellu Str. 8, Riga LV-1002, Latvia

E-mail: ss07036@latnet.lv

[2] *Institute for Electrothermal Processes, Leibniz University of Hannover*

Wilhelm-Busch Str. 4, Hannover D-30167, Germany

Induction crucible furnaces (ICF) that ensure contact less control of hydrodynamic (HD) alloy stirring, temperature and free surface shape are widely applied in metallurgical industry. In such type of furnaces the behaviour of free surface shape might be slightly unsteady due to furnace parameter changes and turbulent HD oscillations.

In this work the model and numerical approach for free surface dynamics calculation in simplified 2D axially symmetric consideration, as well as, in 3D, is developed and its verification is performed by validation of free surface oscillation frequency and comparison of quasi steady state meniscus to other models [1] and appropriate experimental data [2].

The calculation is arranged by means of ANSYS Classic for EM calculation, ANSYS/CFX for HD VOF (Volume of Fluid) calculation and their external coupler (V. Geza, UL) [3].

Transferring free surface keypoints and elementary polygons from CFX to ANSYS Classic a self written filter procedure is performed in order to avoid generation of degenerate surface elements that cause errors in ANSYS Classic mesher. These procedures are aimed mainly on close keypoint merging, polygon reconstruction and further decomposition to equilateral triangles.

Upon imported shape of meniscus the mesh is regenerated and EM calculation is performed. Obtained Lorentz force density is interpolated on fixed CFX mesh and HD calculation is performed for sufficiently small time step. During this time step the change of free surface shape is so small that Lorentz force can be considered constant. By the end of timestep calculation the new free surface shape is obtained and used for Lorentz force recalculation. Such external component coupling ensures free surface dynamics computation.

Appropriate timestep for 3D calculation is chosen by tracing free surface point oscillations on symmetry axis in 2D calculation at different timesteps. The greatest timestep for which the difference in 2D dynamics is below specified criterion is considered sufficiently small.

On the basis of obtained results the analysis of steady state free surface dependence on inductor current and frequency is discussed.

## REFERENCES

[1] O. Pesteanu, E. Baake. Contribution to the simulation of free surface flows in EM field. In: *Proc. of the 54th Intern. Conference on Information Technology and Electrical Engineering, Ilmenau, Germany*, 2009, 361 – 362.

[2] M. Kirpo. *Modelling of turbulence properties and particle transport in recirculated flows*. PhD Thesis, University of Latvia, 2008.

[3] B. Niemann, B. Nacke, V. Geza, A. Jakovics. Simulation of the 3d mass transfer of molten glasses in the inductor-crucible. In: *Proc. of the 6th Intern. Conference on Electromagnetic Processing of Materials, Dresden, Germany*, 2009, 525 – 528.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# COUPLING BETWEEN FLUID AND STRUCTURE MODELS IN REDUCED ORDER AEROELASTIC SYSTEM[1]

WITOLD STANKIEWICZ, ROBERT ROSZAK and MAREK MORZYŃSKI

*Institute of Combustion Engines and Transport, Poznan University of Technology*

Piotrowo 3, 60-965 Poznań, Poland

E-mail: stankiewicz@stanton.ice.put.poznan.pl

The Fluid-Structure Interaction (FSI) plays an important role in the design process in aerospace industry and civil engineering. The deformations of the structure, caused by (changing) aerodynamic forces induced by the flow, might lead to dangerous phenomena like flutter or buffeting.

Computational FSI analysis allows the prediction of such phenomena before performing expensive, time-consuming and potentially destructive experiments like wind-tunnel simulations, ground- and flight-tests. While the progress in aerospace industry is invoked by the reduction of development and certification time, the aeroelastic analyses are getting more and more important.

Due to large sizes of computational meshes, needed to cover all relevant vertex scales, high-fidelity CFD computations (like DNS, RANS, LES and hybrid RANS-LES methods) require large amounts of computer power, memory and time.

The aeroelastic analyses are usually performed on non-conforming grids, with different discretizations on structural and fluid side. The interpolation of pressures calculated by CFD solver as well as boundary deformations and velocities calculated by structural solver are also time-consuming.

The approach that allows significant acceleration of Fluid-Structure Interaction algorithm is the model reduction. While low-dimensional structural solvers, using the most important normal modes, are well known and widely applicable for structures satisfying Hooke's law, Reduced Order Models (ROMs) of fluid flow with unsteady boundary conditions are still under development [1; 2].

In this paper the design of Reduced Order Model of the flow with unsteady boundary is presented. The model is based on Arbitrary Lagrangian-Eulerian (ALE) approach [3] and Galerkin projection of (approximated) governing equation onto space spanned by empirical modes, obtained from high-fidelity CFD computations. The issues of transferring data, namely aerodynamic loads and boundary deflections and velocities, between reduced order models of fluid and structure are discussed and demonstrated on the example of aeroelastic simulation of an airfoil.

## REFERENCES

[1] B.R. Noack, K. Afanasiev, M. Morzyński, G. Tadmor and F. Thiele. A hierarchy of low-dimensional models for the transient and post-transient cylinder wake. *J. Fluid Mech.*, **497** : 335–363, 2003.

[2] W. Stankiewicz, M. Morzyński, R. Roszak, B.R. Noack and G. Tadmor. Reduced Order Modelling of a Flow around an Airfoil with a Changing Angle of Attack. *Archives of Mechanics*, **60** (6): 509–526, 2008.

[3] J. Donea, S. Giuliani and J.P. Halleux. An arbitrary Lagrangian-Eulerian finite element method for transient dynamic fluid-structure interactions. *Comput. Meths. Appl. Mech. Engrg.*, **33** 689–723, 1982.

[1] This work was supported by the Polish Ministry of Science under grant no. N N501 225437.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# INVESTIGATION OF CHARACTERISTIC CURVE FOR STURM–LIOUVILLE PROBLEM ON TORUS

ARTURAS ŠTIKONAS

*Institute of Mathematics and Informatics, Vilnius University*

Akademijos 4, LT-08663 Vilnius, Lithuania

E-mail: arturas.stikonas@mif.vu.lt

We investigate the second-order Sturm–Liouville problem with two additional Nonlocal Boundary Conditions(NBC):

$$\mathcal{L}u := -(p(x)u')' + q(x)u = \lambda u, \tag{1}$$
$$\langle k_0, u\rangle = \gamma_0\langle n_0, u\rangle, \quad \gamma_0 \in \mathbb{R}, \tag{2}$$
$$\langle k_1, u\rangle = \gamma_1\langle n_1, u\rangle, \quad \gamma_1 \in \mathbb{R}, \tag{3}$$

where $p(x) \geq p_0 > 0$, $p \in C^1[0,1]$, $q \in C[0,1]$. NBCs depends on two parameters $\gamma_1$ and $\gamma_2$.

Suppose, we know the fundamental system $\{u_0, u_1\}$ for the homogeneous differential equation. We define a determinant and matrix:

$$D(f_1,f_2)[u_1,u_2] := \begin{vmatrix}\langle f_1,u_1\rangle & \langle f_2,w_1\rangle\\ \langle f_1,u_2\rangle & \langle f_2,w_2\rangle\end{vmatrix}, \quad \mathbf{A} = \begin{pmatrix}D(n_0,n_1)[u_1,u_2] & D(n_0,k_1)[u_1,u_2]\\ D(k_0,n_1)[u_1,u_2] & D(k_0,k_1)[u_1,u_2]\end{pmatrix}.$$

We call the solution of equation

$$D(n_0,n_1)[\boldsymbol{u}]\gamma_0\gamma_1 - D(n_0,k_1)[\boldsymbol{u}]\gamma_0 - D(k_0,n_1)[\boldsymbol{u}]\gamma_1 + D(k_0,k_1)[\boldsymbol{u}] = 0.$$

a *Characteristic Curve* for problem (1)–(3). We find condition for existence of zero eigenvalue in the parameters space and classified Characteristic Curves in the plane and extended plane is described as torus.

LEMMA 1. *On torus Characteristic Curve for problem* (1)–(3) *can be one of the following three types:*

(1) *If* $\mathbf{A} \in GL_2(\mathbb{R})$ *then the curve is homeomorphic to a circle, and this curve winds around the torus one time (one time in one direction and one in the other direction);*

(2) *If* $\mathbf{O} \neq \mathbf{A} \notin GL_2(\mathbb{R})$ *then the curve is the union of two circles (strictly 'latitudinal' and strictly 'longitudinal') with one common point;*

(3) *Otherwise (i.e.,* $\mathbf{A} = \mathbf{O}$*) the curve is whole torus.*

**REFERENCES**

[1] R. Čiegis, A. Štikonas, O. Štikonienė and O. Suboč. Stationary Problems with Nonlocal Boundary Conditions. *Math. Model. Anal.*, **6** (2):178–191, 2001.

[2] A. Štikonas. Investigation of Characteristic Curve for Sturm–Liouville Problem with Nonlocal Boundary Conditions on Torus. *Math. Model. Anal.*, **16** (1):1–22, 2011.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON COMPLEX AND NEGATIVE EIGENVALUES OF DIFFERENCE OPERATOR WITH NONLOCAL CONDITIONS

OLGA ŠTIKONIENĖ[1,2] and MIFODIJUS SAPAGOVAS[1]

[1] *Vilnius University, Institute of Mathematics and Informatics*

Akademijos 4, LT-08663, Vilnius, Lithuania

[2] *Vilnius University, Department of Mathematics and Informatics*

Naugarduko 24, LT-03225, Vilnius, Lithuania

E-mail: olga.stikoniene@mif.vu.lt, mifodijus.sapagovas@mii.vu.lt

The investigation of the structure of spectrum of the difference operator is useful for many problems with nonlocal conditions. We consider the Sturm-Liouville problem for the differential operator with integral conditions

$$\begin{aligned} &\frac{d^2u}{dx^2} + \lambda u = 0, \\ &u(0) = \gamma_1 \int_0^1 \alpha(x)u(x)dx, \\ &u(1) = \gamma_2 \int_0^1 \beta(x)u(x)dx, \end{aligned}$$

where $\alpha(x), \beta(x)$ are weights and boundary conditions depend on the parameters $\gamma_1$ and $\gamma_2$. The eigenvalue problem for the corresponding difference operator is investigated, too.

The existence of a negative eigenvalue (or a complex one with negative real part) may lead to unstability of the difference scheme for the parabolic problem [1; 2] or to divergence of the iteration method for the stationary problem [3].

We have studied the dependence of conditions, where such eigenvalues exist, on the coefficients of nonlocality $\gamma_1, \gamma_2$ and weight functions $\alpha(x), \beta(x)$.

**REFERENCES**

[1] A.V. Gulin and V.A. Morozova. On a family of nonlocal difference schemes. *Diff. Equ.*, **45** (7):1020-1033, 2009.

[2] F. Ivanauskas and T. Meškauskas and M. Sapagovas. Stability of difference schemes for two-dimensional parabolic equations with non-local boundary conditions. *Appl. Math.Comp.*, **215** (7):2716–2732, 2009.

[3] M. Sapagovas and A. Štikonas and O. Štikonienė. Alternating-direction method for Poisson equation with weighted coefficients in integral condition. *Differ. Equ.*, 2011. (submitted)

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# MODELLING, NUMERICS AND ANALYSIS OF FIBER SUSPENSION FLOWS[1]

ULDIS STRAUTIŅŠ

*Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: uldis.strautins@lu.lv

The rich variety of dynamics observed in fiber suspension flows emerges from the complex interplay among the fibers and the fluid phase subjected to external forces and poses considerable challenge to the modeller. In this talk, we focus on the flow of a suspension that admits hydrodynamical description, the fiber volume fraction ranging from dilute to concentrated regimes. Both flows in the bulk of the material and in the vicinity of a flat solid wall are considered.

A host of new models concerning the fiber suspension rheology has been proposed in the recent years, see e.g. [1; 2]. We review some current trends in the modelling and present a novel model for the orientation dynamics of fibers near a solid wall.

Significant advances have been made in computational rheology enabling direct numerical simulations of PDEs in configuration spaces of high dimension using the Proper Generalized Decomposition method, see [3]. This approach circumvents the closure problem and hence is of great interest for simulation of fiber suspension flows.

We consider the mesoscale rheological system of PDEs in a formulation that includes most of the considered models and estabilish certain well-posedness results.

**REFERENCES**

[1] J. H. Phelps and C. L. Tucker. An anisotropic rotary diffusion model for fiber orientation in short- and long-fiber thermoplastics. *Journal of Non-Newtonian Fluid Mechanics*, **156** 165–176, 2009.

[2] A. Latz, U. Strautins and D. Niedziela. Comparative numerical study of two concentrated fiber suspension models. *Journal of Non-Newtonian Fluid Mechanics*, **165** 764–781, 2010.

[3] A. Ammar, B. Mokdad, F. Chinesta and R. Keunings. A New Family of Solvers for Some Classes of Multidimensional Partial Differential Equations Encountered in Kinetic Theory Modeling of Complex Fluids. *Journal of Non-Newtonian Fluid Mechanics*, **139** 153–176, 2006.

[1]This work is partially supported by the projects 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 of the European Social Fund.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ALGORITHMS FOR DECOMPOSITION OF SIMPLE POLYGON WITH HOLES

OLGA SUBOČ and DANAS MOTIEJAUSKAS

*Vilnius Gediminas Technical University*

Sauletekio av. 11, Vilnius, Lithuania

E-mail: danas.motiejauskas@gmail.com, os@vgtu.lt

Modern geografic information systems (GIS) are videlly used in various areas of science and industry. Most of data is saved as polygons (edges of areas, forests, lakes, etc.). This data can be very precise, and number of vertices in these polygons can be huge. A software used to operate geographical data often has various restrictions, for example, it is impossible to interpret holes in polygon, or a maximal number of vertices of polygon is fixed.

In this article we deal with decomposition of simple polygon with holes into components so that every piece does not exceed some defined number of vertices. Existing studies and algorithms for polygon decomposition are analyzed. We propose two modifications of polygon triangulation and approximate convex decomposition algorithms. The complexity analysis of both algorithms is done. It is shown, that the complexity of modified polygon decomposition algorithm is $O(n \log n)$, and complexity of modified approximate convex decomposition algorithm in best case is $O\left(n\left(\frac{n-2}{V_{max}-2}\right)\right)$, and in worst case $O(n^2)$. In the experimental part of the work results of computing experiments are presented, analyzed and compared with the theoretical complexity bounds.

**REFERENCES**

[1] Keil, J. M. Polygon decomposition. In: *J.-R. Sack and J. Urrutia(Ed.), Handbook of Computational Geometry, Amsterdam:*, Elsevier Science Publishers, 1998, 491-518.

[2] Lien, J.-M.; Amato, N. M. Approximate convex decomposition of polygons. In: *Proc. 20th Annual ACM Symposium on Computational Geometry (SoCG).*, New York, 2004, 17-26.

[3] Chazelle, B. Triangulating a simple polygon in linear time. In: *Discrete & Computational Geometry: vol. 6, issue 5. New York:*, Springer-Verlag, 1991, 63-133.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# DETAILS OF VOLUME FREE ELECTRON LASER NONLINEAR DYNAMICS

SVETLANA SYTOVA

*Research Institute for Nuclear Problems, Belarusian State University*

Bobruiskaya 11, 220030 Minsk, Belarus

E-mail: sytova@inp.bsu.by

From the middle of the 20th century after creation of a wide class of electronic devices such as travelling wave tubes, backward wave tubes, orotrons, free electron lasers etc. the epoch of rapid growth of vacuum electronics was originated. The principles of functioning of such devices are based on radiation of bunches of charged particles moving over the surface or through the slow-wave system (resonator).

A new type of vacuum electronic devices proposed in the '80s was Volume Free Electron Laser (VFEL) [1] where radiation of bunches of charged particles moving through resonator is obtained under multi-wave volume (non-one-dimensional) distributed feedback (VDFB) in conditions of dynamical diffraction of electromagnetic radiation. VFEL can be created in different wavelenght ranges and on different radiation mechanisms. First lasing of VFEL in mm wavelength range [2] put the beginning of its experimental development.

The system of nonlinear equations used for VFEL modelling was obtained from Maxwell equations in the slowly-varying envelope approximation [3]. Electron beam dynamics is described using method of averaging over initial phases of electrons. Equation describing phase of electron beam is more complicated than usually used in simulation of different electronic devices because it was taken into consideration as initial phase of an electron not only the moment of time $t_0$ of an electron entrance in resonator at $z = 0$ (as usual) but also transverse spatial coordinate in this moment. The full system of equations is solved numerically [3].

In our investigations it was shown that VFEL is a dynamical system with multiple bifurcation points and chaotic dynamics [4]. Here some details of VFEL nonlinear dynamics are discussed.

During numerical experiments it was proved one of the main VFEL features. Namely due to VDFB the mode discrimination leads to parasitic mode suppression. This becomes apparent in different types of chaotic regimes for passing and diffracted electromagnetic waves inside the VFEL resonator. It was obtained analytically and numerically solution of the system for VFEL steady-state regimes and some periodic regimes. Such regimes are observed in ribbon of parameters along the threshold conditions of VFEL generation.

## REFERENCES

[1] V. Baryshevsky and I. Feranchuk. Parametric beam instability of relativistic charged particles in a crystal. *Phys. Let. A*, **102** 141–144, 1984.

[2] V. Baryshevsky et al. First lasing of a volume FEL (VFEL) at a wavelength range 4-6 mm. *Nucl. Instr. Meth. Phys. Res.*, **A483** 21–24, 2002 .

[3] K. Batrakov and S. Sytova. Modelling of Volume Free Electron Lasers. *Computational Mathematics and Mathematical Physics*, **45** 4:666–676, 2005.

[4] S. Sytova. Some aspects of chaotic lasing in volume free electron lasers. *Nonlinear Phenomena in Complex Systems*, **12** 37-45, 2009.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# TIME CHANGES IN ASSET PRICE MODELING

DEAN TENENG

*Institute of Statistics, Tartu University*

J Liivi 2, Tartu, 50409, Estonia

E-mail: teneng@ut.ee

Many popular models in Finance are based on time-changed Brownian motion. Due to the self-similarity property of Brownian motion [1],

$$B(tc^2) = cB(t), c > 0 \tag{1}$$

the qualitative features of asset price trajectories should not change if time scale changes. By comparing the descriptive statistics of ten Estonian companies, it is shown that this is not the case, prompting the need for subordinators and absolute continuous time changes [2; 3].

**REFERENCES**

[1] Peter Mörters and Yuval Peres. *Brownian Motion*. http://www.stat.berkeley.edu/ peres/bmbook.pdf, accessed 15 March 2011.

[2] E. Barndorff-Nielsen, and J. Schmiegel . Time change and Universality in Turbulence and Finance. *Draft*, **1** accessed 30 March 2011.http://old.thiele.au.dk/turbulence/abstracts/ongoing.draft.4.pdf

[3] M. Winkel, and A. Veraart . Time change. *Draft*, **1** accessed 15 Feb 2011.http://www.stats.ox.ac.uk/ winkel/winkel15.pdf

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SIMULATION OF HIGH ENERGY HALO PARTICLE EXTRACTION FROM ACCELERATOR USING BENT CRYSTAL

VICTOR TIKHOMIROV and ALEXEI SYTOV

*Research Institute for Nuclear Problems, Belarusian State University*

Bobruiskaya 11, 220030 Minsk, Belarus

E-mail: vvtikh@mail.ru, alex_sytov@mail.ru

Channeling is an effect of charge particles penetration through a monocrystal parallel to its atomic axis or planes. It is better to use planar channeling for manipulation high energy particles.

This work is devoted to simulation of one of possible applications of channeling in bent crystals. This is the capability of bent crystals to deflect a particle beam in high energy accelerators. For example the collimation system including harmful halo particle absorbers becomes more effective using such crystal modifications. One possible application of this enhanced collimation system is the Large Hadron Collider (LHC) luminosity upgrade.

The collimation system based on bent crystal is simulated using the Monte-Carlo method. This simulation includes two stages: interplanar potential motion of ultrarelativistic particle in the silicon crystal and its motion in accelerator. Initial conditions (particles coordinates and angles) are simulated before the first particle pass through crystal. Also nuclear and electron scattering were simulated at each step. One can evaluate the particle trajectory in crystal solving the second-order linear differential equation obtained from the condition of transverse energy conservation. This equation is solved using the fourth-order Runge-Kutta method.

Consideration of particle dynamics in crystal as motion in plain-averaged interplanar potential allows to combine all different kinds of motion (channeling, volume reflection and motion in amorphous-like substance). Also it allows to simulate a narrow plane cut that can increase the channeling efficiency [1]. A motion in the synchrotron-type accelerator is connected with betatron oscillations which are described by the Hill equation. Its solution can be expressed in terms of the Courant-Snyder coefficients known for each accelerator section. So, the particle trajectory in accelerator can be obtained analytically.

A computer program written on Delphi realizes all items cited above. It requires beam and crystal parameters and allows to obtain coordinate and angle distributions at the end of the crystal and at the absorber in accelerator, as well as phase space at any step in crystal and the channeling and collimation efficiency. This program was tested on results of the UA9 experiment at SPS accelerator (CERN) [2]. A good agreement with experimental dependence of collimation efficiency from the crystal orientation with respect to beam incident angle was achieved. Also most of the results of simulation [1], [2] were reproduced with a good agreement. So, our computer program can be used for simulation of technique of improving crystal channeling efficiency and for forthcoming proposals of collimation system for LHC luminosity upgrade.

**REFERENCES**

[1] V.Tikhomirov. A technique to improve crystal channeling efficiency of charged particles. *JINST*, **2** P08006, 2007.

[2] W. Scandale et al. First results on the SPS beam collimation with bent crystals. *Phys. Let.*, **B692** 78–82, 2010 .

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# NEW ITERATIVE METHOD FOR INVERSION OF OPERATORS CORRESPONDING TO CONSERVATIVE FINITE-DIFFERENCE SCHEME FOR THE MULTIDIMENSIONAL SHRÖDINGER EQUATION WITH COMPLEX BOUNDARY CONDITIONS

VYACHESLAV A. TROFIMOV[1], ARTEM V. ROZANTSEV[1], ZHONGYI HUANG[2] and HOUDE HAN[2]

[1]*Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University*

[1]Leninskie Gory,119992, Moscow Russia

[2]*Department of Mathematical Sciences,Tsinghua University*

[2] Beijing, 100084, China

E-mail: [1]vatro@cs.msu.ru

In this report we propose (see as well [1] )very simple iterative method of solving the conservative finite-difference scheme for multidimensional Shrödinger equation with complex boundary conditions. Developing of such method causes by a necessity of using the non-reflecting boundary conditions for multidimensional Shrödinger equation. In this case an application of FFT method for solution of this problem is impossible. The other way of solution concludes in using the split-step method. As it is well-known this method is high effective for linear problem. Nevertheless, an application of this method for solution of the Shrödinger equation leads to realization of non-conservative finite-difference schemes. Hence, developing the method for inversion of the corresponding finite-difference operators in multidimensional case without loss of the conservatism property of finite-difference scheme is a modern problem. We demonstrate our two-steps iterative method at consideration of 2D nonlinear Shrödinger equation

$$\frac{\partial u}{\partial t} - \imath D_x \frac{\partial^2 u}{\partial x^2} - \imath D_y \frac{\partial^2 u}{\partial y^2} + \imath(V(x,y)+\beta)u + \imath\gamma|u|^2 u = 0, t > 0, 0 < x < L_x, 0 < y < L_y.$$

Let us write for this equation the conservative finite-difference scheme with the following iterative method:

$$\frac{\overset{s+1}{\hat{U}} - U}{\tau} - \imath D_x \Lambda_{\bar{x}x} \overset{s+1}{\underset{0.5}{U}} - \imath D_y \Lambda_{\bar{y}y} \overset{s}{\underset{0.5}{U}} + \imath(V+\beta) \overset{s+1}{\underset{0.5}{U}} + \imath\gamma |\overset{s}{\underset{0.5}{U}}|^2 \overset{s}{\underset{0.5}{U}} = 0, \overset{0.5}{U} = 0.5(\hat{U}+U)$$

$$\frac{\overset{s+2}{\hat{U}} - U}{\tau} - \imath D_x \Lambda_{\bar{x}x} \overset{s+1}{\underset{0.5}{U}} - \imath D_y \Lambda_{\bar{y}y} \overset{s+2}{\underset{0.5}{U}} + \imath(V+\beta) \overset{s+2}{\underset{0.5}{U}} + \imath\gamma |\overset{s+1}{\underset{0.5}{U}}|^2 \overset{s+1}{\underset{0.5}{U}} = 0, \overset{0.5}{|U|^2} = 0.5(|\hat{U}|^2 + |U|^2).$$

This paper was partly supported by Russian Foundation for Basic research (N 10-07-90004-Bel-a).

**REFERENCES**

[1] A. V. Rozantsev. Research work done on the 4th year of education. *Lomonosov Moscow State University*, **2011** (in Russian).

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ABOUT THE EFFICIENCY OF USING THE ROSENBROCK METHOD FOR NONLINEAR OPTIC PROBLEMS

VYACHESLAV A. TROFIMOV and EVGENII M. TRYKIN

*Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University*

Leninskie Gory,119992, Moscow Russia

E-mail: vatro@cs.msu.ru

In this report we investigate an efficiency of solving the 1D nonlinear Shrödinger equation or set of such equations with using the Rosenbrock method. The application of this method for 1D nonlinear Shrödinger equation is proposed in [1]. This method allows to solve a nonlinear equation without using any iterative method. Obviously, it is an advantage of this method. Nevertheless, questions both of accuracy of this method and of its preservation of conservation laws are not discussed by anybody. Keeping this in mind, we consider the following nonlinear Shrödinger equation

$$\frac{\partial A}{\partial t} + \imath D\frac{\partial^2 A}{\partial z^2} + \imath\gamma V(z)A + \imath\alpha|A|^2A = 0, t > 0, 0 < z < L_z.$$

According to the Rosenbrock method, a real function is represented into real and imaginary parts. In our case the complex amplitude $A$ contains already the real and imaginary parts. Hence, first of all we rewrite the nonlinear Shrödinger equation as a set of equations with respect to real and imaginary parts. Let us remind that the solution of equation (see the example below) on the upper time level by applying the Rosenbrock method is calculated in the following way:

$$\frac{\partial U(t)}{\partial t} = G(U), \qquad \hat{U} = U + \tau Rek, \qquad (E - \beta\tau G_U)k = G(U).$$

Here, $U$ is a required function, $G(U)$ is a right part of the equation. Function $k$ is a complex one. Because of this, for our Shrödinger equation the two complex functions which are similar to the function $k$ are used.

It should be stressed that Rosenbrock method is conservative conditionally. To get unchangeable value of invariants during chosen time interval it is necessary to choose the mesh steps in certain way. The value of mesh steps depends on parameters $D, \alpha, \gamma$ and time interval. The other important feature of this method is fast increasing the time of computation with decreasing of mesh steps. Starting from some value of mesh steps the conservative finite-difference scheme becomes many times effective in comparison with Rosenbrock method.

We discuss also the other important question for practice is an application of the Rosenbrock method for solving the set of nonlinear Shrödinger equation.

This paper was partly financially supported by Russian Foundation for Basic research (grant number 10-07-90004-Bel-a).

**REFERENCES**

[1] N.N. Kalitkin, E.A. Alshina and P.V. Koryakin. Diagnostics of singularities of the exact decision at calculations with accuracy control. *J. of Computational Mathematics and Mathematical Physics*, **45** (10):1837 - 1847, 2005.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# PARALLEL FULLY IMPLICIT ALGORITHM FOR SYSTEMS OF PARABOLIC EQUATIONS ON GRAPHS

NATALIJA TUMANOVA and RAIMONDAS ČIEGIS

*Vilnius Gediminas Technical University*

Saulėtekio al. 11, LT-10223 Vilnius, Lithuania

E-mail: tumnat@gmail.com

We consider a reaction-diffusion parabolic problem formulated on the edges of the graph. The well-known examples of such type of equations are given by neuron simulation models which are based on the Hodgkin–Huxley (HH) reaction–diffusion system. The equations are coupled due to some conjugation conditions at the branch points.

We investigate the fully implicit finite–difference scheme, which basic template is obtained by using the implicit backward Euler scheme to approximate the system of differential equations. The system of nonlinear equations can be linearized with the Picard type iterations. The approximation of the conservation equations at the branching points is similar to the approximation of the boundary conditions of the second and third type. Although these points complicate the convergence analysis of the whole discrete scheme [1; 2], the convergence rate of $(\tau + h^2)$ is proven for the linear problem using the modified maximum principle [3].

The investigated parallel algorithm applies data distribution paradigm to distribute the discrete problem among processors. Metis tool is applied for a balanced distribution of local tasks. The large system of linear equations formed at every time step can be reduced to the size of number of branching points using modified factorization algorithm [4]. This step is implemented locally on each processor. Then the reduced system of equations is solved in parallel using PCG method. Due to the known structure of the domain, every processor knows exactly the structure of the sparse linear matrix, and this reduces the communication costs of the step. When solutions on branching points are computed, solutions on edges are calculated using backward factorization algorithm.

The results of computational experiments are presented and the efficiency of the proposed parallel algorithm is investigated.

**REFERENCES**

[1] R. Čiegis and N. Tumanova. Finite-difference schemes for parabolic problems on graphs. *Lith. Math. Journal*, **50** (2):164–178, 2010.

[2] R. Čiegis and N. Tumanova. Parallel predictor-corrector schemes for parabolic problems on graphs. *Computational Methods in Applied Mathematics*, **10** (3):275–282, 2010.

[3] A. A. Samarskii. *The Theory of Difference Schemes*. Nauka, Moscow, 1989. (in Russian)

[4] M. Mascagni. A parallelizing algorithm for computing solutions to arbitrarily branched cable neuron models. *J. Neurosci. Methods*, **36** (1):105–114, 1991.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON THE CATEGORY OF $L$-VALUED BORNOLOGICAL SPACES[1]

INGRIDA UĻJANE[1,2] and ALEKSANDRS ŠOSTAKS[1,2]

[1] *Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga LV-1002, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: uljane@mail.com, sostaks@lu.lv

In our talk at the previous conference "Mathematical Modelling and Analysis 2010" (Druskininkai, May 2010) we have introduced the concept of an $L$-valued bornology where $(L, \leq, \bigwedge, \bigvee, *)$ is a cl-monoid, in particular (in case $* = \wedge$) $L$ is a complete infinitely distributive lattice. Namely, by an $L$-valued bornology on a set $X$ we mean a mapping $\mathcal{B} : 2^X \to L$ such that

1) $\forall x \in X \quad \mathcal{B}(\{x\}) = 1$;

2) If $U \subset V \subset X$ then $\mathcal{B}(V) \leq \mathcal{B}(U)$;

3) $\forall\, U, V \subset X \quad \mathcal{B}(U \cup V) \geq \mathcal{B}(U) * \mathcal{B}(V)$.

A mapping $f : (X, \mathcal{B}_X) \to (Y, \mathcal{B}_Y)$ where $(X, \mathcal{B}_X)$ and $(Y, \mathcal{B}_Y)$ are $L$-valued bornological spaces is called bounded if $\mathcal{B}_X(A) \leq \mathcal{B}(f(A))$ for every $A \in 2^X$. Some properties of the category of $L$-valued bornological spaces and their bounded mappings were also discussed in our talk at the International Conference on Topology and Applications in Nafpaktos, Greece in 2010, see [1]

In the present talk we continue the study of $L$-bornological spaces and their bounded mappings under the additional assumption that $L$ is equipped with an order reversing involution $^c : L \to L$. In particular, we present a construction of an $L$-valued bornology $\mathcal{B} : 2^X \to L$ from a non-decreasing (by inclusion) family of usual, that is crisp, bornologies $\mathcal{C} = \{C_\alpha, \alpha \in L\}$ on a set $X$. This construction is given by the equality

$$\mathcal{B}(A) = \bigvee \{\alpha^c \mid A \in C_\alpha\} \quad \forall A \in 2^X.$$

The lattice structure and the categorical properties of this construction will be discussed.

*Remark 1.* Note that in case $L = \{0, 1\}$ our concept of an $L$-valued bornology turns into the well known concept of a bornology introduced by S.-T. Hu [2] and thoroughly studied recently by different authors. Note also that the *crisp* bornological-type structure on the family $L^X$ of $L$-subsets of a set $X$ was introduced and studied in [3].

**REFERENCES**

[1] I. Uļjane and A. Šostak. On bornological-type structures in the context of $L$-fuzzy sets. In: *Abstracts of 2010 International Conference on Topology and Applications, Nafpaktos, Greece*, Contribution of the Technological Educational Institute of Messolonghi S.D. Iliadis and D.N. Georgiou (Eds.), 2010, 226 – 229.

[2] S.-T. Hu. *Introduction to General Topology.* Holden-Day, San-Francisco, 1966.

[3] M. Abel and A. Šostak. Towards the theory of $L$-bornological spaces. *Iranian Journal of Fuzzy Systems*, **8** (1): 19-28, 2011.

[1] This work was partly supported by ESF project Nr. 2009/0223/1DP/1.1.1.2.0/09/APIA/VIAA/008 and by the Latvian Science Foundation project 09.1570.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# SPLITTING SCHEMES FOR HYPERBOLIC HEAT CONDUCTION EQUATION

PETR VABISHCHEVICH

*Nuclear Safety Institute, Russian Academy of Sciences*

52 B. Tul'skaya, Moscow 115191, Russia

E-mail: vab@ibrae.ac.ru

Linear parabolic theories of diffusion and heat conduction are based on the Fick and Fourier laws, respectively, and predict an infinite speed of propagation. In this case, the amplitude of propagating perturbations decreases exponentially with the distance and the infinite speed of perturbations can often be ignored. Nevertheless, in many applied problems the wave nature of heat transfer should be taken into account. Various corrections have been proposed for parabolic heat conduction models in order to eliminate the paradox of infinite speed of perturbation propagation.

The standard parabolic heat conduction model is based on the explicit representation of the heat flux through the temperature gradient. The hyperbolic heat conduction model includes an additional term with the time derivative for the heat flux which is proportional to the relaxation tensor. More general models in addition includes the relaxation of the temperature gradient. Separate attention should be given to the hyperbolic model of convection-diffusion for moving media.

Two possibilities can be highlighted in constructing computational methods for the approximate solution of hyperbolic heat transfer problems. The first is connected with the transition from a system of the first order evolutionary equations for the temperature and heat flux to a single hyperbolic equation of second order. In contrast to the standard parabolic equation of heat conduction there does present a term with the second time derivative. The second possibility is based on the usage of the initial scalar-vector system of equations.

Much attention is paid to the construction of the additive schemes (splitting schemes) for approximate solving initial-boundary value problems for multi-dimensional partial differential equations. Transition to a sequence of more simple problems allows to construct, for example, economical difference schemes - schemes based on the splitting with respect to spatial variables. In some cases it is reasonable to perform splitting with respect to subproblems of different nature - splitting in physical processes. At present regionally-additive schemes (domain decomposition methods) are actively discussed. These schemes are oriented to the construction of computational algorithms for parallel computers.

In the present work there are constructed splitting schemes with respect to spatial variables for the approximate solving the equation of hyperbolic heat conduction. Unconditionally stable locally one-dimensional difference schemes are constructed here both for a single heat conduction equation and for the system of equations based on the temperature and heat flux as unknowns.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# CORDIAL VOLTERRA INTEGRAL EQUATIONS OF THE FIRST KIND

GENNADI VAINIKKO

*Institute of Mathematics, University of Tartu*

J. Liivi 2, 50409 Tartu, Estonia

E-mail: gennadi.vainikko@ut.ee

Consider the equation $V_\varphi u = f$, where $(V_\varphi u)(t) = \int_0^t t^{-1}\varphi(t^{-1}s)u(s)ds$, $0 < t \le T$, $\varphi \in L^1(0,1)$. The spectrum of $V_\varphi$ as an operator in the space $C^m = C^m[0,T]$ is described in [1, 2]. This enables to establish criteria for the existence and boundedness of the inverse $V_\varphi^{-1}$ as an operator from $C^{m+k}$ to $C^m$, $m \ge 0$, $k \ge 1$. In certain cases, $\|V_\varphi^{-1}\|$ can be effectively estimated, for instance:

**Theorem 1.** *Let $\varphi \in L^1(0,1)$, $\widehat{\varphi}(0) := \int_0^1 \varphi(x)dx > 0$, $x\varphi' \in L^1(0,1)$, and let $x\varphi'(x)+\alpha\varphi(x) \ge 0$ $(0 < x < 1)$ for an $\alpha < 1$. Then $V_\varphi^{-1} \in \mathcal{L}(C^{m+1}, C^m)$ exists, and*

$$\|V_\varphi^{-1} f\|_{C^m} \le \frac{1}{(1-\alpha)\widehat{\varphi}(0)} \|tf' + (1-\alpha)f\|_{C^m} \text{ for } f \in C^{m+1},\ m \ge 0.$$

*The claim remains to be true if condition $x\varphi' \in L^1(0,1)$ is relaxed to the form $x\varphi' \in L^1(0,1-\varepsilon)$ for any $\varepsilon > 0$, and $\lim_{x\to 1} \varphi(x) = \infty$, $\lim_{x\to 1}(1-x)\varphi(x) = 0$.*

More complete results are obtained using instead of $C^m$ the weighted spaces

$$C_\star^{m,r} = \left\{ u \in C^m(0,T] : \lim_{t\to 0} t^{k-r} u^{(k)}(t) \text{ exists for } k = 0, 1, ..., m \right\},$$

$$\|u\|_{C_\star^{m,r}} = \max_{0\le k\le m} \sup_{0<t\le T} t^{k-r}|u^{(k)}(t)| \quad m \ge 0, \quad r \in \mathbb{R}.$$

The power functions $t^p$, $p \ge r$, are eigenfunctions of $V_\varphi$ in $C_\star^{m,r}$, $m \ge 0$. This enables to design approximate and exact solvers of equation $V_\varphi u = f$. Theorem 1 and further results of this type tell us, how precise a generalized polynomial approximation $f_N$ to $f$ should be constructed in order to achieve a desired accuracy of the approximation $V_\varphi^{-1} f_N$ to $V_\varphi^{-1} f$; if $f$ can be expanded into a generalized power series around 0 then also $V_\varphi^{-1} f$ has the form of a generalized power series of the same convergence radius.

**REFERENCES**

[1] G. Vainikko. Cordial Volterra integral equations 1. *Numer. Funct. Anal. Optim.*, **30:** 1145–1172, 2009.

[2] G. Vainikko. Cordial Volterra integral equations 2. *Numer. Funct. Anal. Optim.*, **31:** 191–219, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# RECENT TRENDS IN ROBUST STATISTICS

JĀNIS VALEINIS

*Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: valeinis@gmx.net

A lot of classical statistical tests, estimation procedures rely on the assumption of normality of data. Robust statistics aim to derive methods for statistical inference which works also when the classical assumptions are slightly violated. Mainly these procedures tend to work also in the presence of outliers. Whether an estimator is robust or not can be detected by influence function derivation. Well known is the class of M-estimators, which include the famous robust Huber M-estimator. Recently Hampel *et al.* [1] introduced smoothed M-estimators, which appear to perform better than its empirical coutnerparts. There are many recent results for robust regression problems, such as: robust estimation of scale function in heteroscedastic nonparametric models [2], robust binary regression [3], robust test for homoscedasticity in nonparametric regression [4] etc. We will discuss these and related recent results in robust statistics and show some implementation and simulations.

**REFERENCES**

[1] F. Hampel, C. Hennig, E. Ronchetti. A smoothing principle for the Huber and other location M-estimators. *Computational Statistics and Data Analysis*, **55** (1):324–337, 2011.

[2] G. Boente and M. Ruiz, and R.H. Zamar. On a robust local estimator for the scale function in heteroscedastic nonparametric regression. *Statistics & Probability Letters*, **80** (15-16):1185–1195, 2010.

[3] S. Hosseinian and S. Morgenthaler. Robust binary regression. *Journal of Statistical Planning and Inference*, **141** (4):1497-1509, 2011.

[4] H. Dette and M. Marchlewski. A robust test for homoscedasticity in nonparametric regression. *Journal of Nonparametric Statistics*, **22** (6):723-736, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# RESAMPLING METHODS IN THE FREQUENCY DOMAIN

AGRIS VASELĀNS and JĀNIS VALEINIS

*Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: av07050@lu.lv, valeinis@gmx.net

Spectral analysis of time series has become an actual topic over the last few decades due to the development of numerous new approaches in non-parametric spectrum estimation. Resampling methods in the frequency domain use the terms of spectral analysis in order to devise the limiting distribution for different kinds of statistics, especially to obtain critical values for hypothesis testing. Some bootstrap methods such as periodogram bootstrapping etc. have been proposed lately [2]. Two kinds of approaches have been used. First one resamples the periodogram, while the second one proposes to resample the Fourier coefficients. Recently a new powerful time series bootstrap scheme, called the Time Frequency Toggle (TFT) Bootstrap, has arisen with applications in the change point analysis [1; 3]. TFT bootstrap procedure uses the second approach, which can be easily realised using Fast Fourier transform. In this work we give an insight into some applications of the resampling methods in the frequency domain, in particular change point analysis, and compare different approaches described above using some simulation study.

**REFERENCES**

[1] C. Kirch, N. D. Politis. TFT-Boostrap: Resampling time series in the frequency domain to obtain replicates in the time domain. *The Annals of Statistics*, 2011, To appear.

[2] E. Paparoditis. *Frequency domain bootstrap for time series.* Empirical process techniques for dependent data, Birkhauser, Boston. 2002

[3] C. Kirch. Resampling in the frequency domain of time series to determine critical values for change-point tests.. *Statistics and Decision*, **25** (3):237-261, 2007.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON SOME MODIFICATIONS OF FUZZY TRANSFORMS

IRINA VAVILCENKOVA

*Department of Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: irina.vavilcenkova@gmail.com

The technique of fuzzy transforms proposed by I. Perfilieva (see e.g. [1] - [3]) is based on a fuzzy partition of an interval $[a, b]$ into fuzzy subsets $A_1, A_2, ..., A_n$. For a given mesh $\Delta : \ a = x_1 < x_2 < ... < x_n = b$, continuous functions $A_1, A_2, ..., A_n$ defined on $[a, b]$ form a fuzzy partition if they fulfil the following conditions:

$A_k(x) = 0$ if $x \notin (x_{k-1}, x_{k+1})$ and $A_k(x_k) = 1$, $k = 1, 2, \ldots, n$ (here $x_0 = a$ and $x_{n+1} = b$);

$A_k$ increases on $[x_{k-1}, x_k]$ if $k = 2, 3, \ldots, n$, and decreases on $[x_k, x_{k+1}]$ if $k = 1, 2, \ldots, n-1$;

$\sum_{k=1}^{n} A_k(x) = 1$ for all $x \in [a, b]$.

In this talk we consider splines of degree $m$ and defect 1 with respect to $\tilde{\Delta}$ (here $\tilde{\Delta}$ is an extension of $\Delta$) as basic functions of the fuzzy partition:

$$A_k^m \in S_{m,1}(\tilde{\Delta}), \ k = 1, 2, \ldots, n,$$

and investigate the spline based fuzzy transform $F^m[f] \in \mathbb{R}^n$, whose component $F_k^m[f]$ for a given continuous function $f$ minimizes

$$\Phi_k^m(y) = \int_a^b (f(x) - y)^2 A_k^m(x)\, dx, \quad k = 1, 2, \ldots, n.$$

The talk deals also with the transform $H^m[f] \in \mathbb{R}^n$, components $H_k^m[f]$, $k = 1, 2, ..., n$, of which give minimum to the function

$$\Phi^m(y_1, y_2, ..., y_n) = \int_a^b (f(x) - \sum_{k=1}^{n} y_k A_k^m(x))^2\, dx.$$

We compare approximation properties of two spline based inverse transforms

$$f_{F,m,n} = \sum_{k=1}^{n} F_k^m[f] A_k^m \quad \text{and} \quad f_{H,m,n} = \sum_{k=1}^{n} H_k^m[f] A_k^m,$$

and illustrate them with numerical examples, when $m = 1, 2, 3$.

**REFERENCES**

[1] I. Perfilieva. Fuzzy transforms: Theory and applications. *Fuzzy Sets and Systems*, **157** (8):993–1023, 2006.

[2] I. Perfilieva. Fuzzy transforms and their applications to data compression. In: *Proc. Int. Conf. on FUZZ-IEEE*, Reno, Nevada, USA, 2005, 294–299.

[3] I. Perfilieva. Fuzzy transform: application to reef growth problem. In: *R.B. Demicco, G.J. Klir (Eds.), Fuzzy Logic in Geology*, Academic Press, Amsterdam, 2003, 275–300 .

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# HUBER SMOOTH M-ESTIMATOR

MĀRA VĒLIŅA and JĀNIS VALEINIS

*Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: mara.velina@gmail.com, valeinis@gmx.net

M-estimators cover a broad class of estimators in mathematical statistics, such as the least squares, maximum likelihood and robust location estimators. When introducing the empirical likelihood method, Owen [2] showed that it can be used also for making the inference about M-estimators. One of the most famous M-estimator is the Huber location estimator introduced in [1]. The Huber estimator still attracts researchers, especially from the field of robust statistics. In the recent contribution by Hampel [3] a smoothing principle for M-estimators has been provided and improvement against their non-smoothed counterparts in simulation setting has been demonstrated.

We further analyse the improvements attained through the smoothing of M-estimators by comparing the coverage accuracy of the empirical likelihood confidence intervals of smoothed M-estimators and bootstrap methods using simulations. In particular, the analysis of the Huber M-estimator is carried out. In addition, the two–sample problem for the difference of two Huber M-estimators is analysed.

**REFERENCES**

[1] P. J. Huber. Robust estimation of a location parameter. *Annals of Mathematical Statistics*, **35** (1):73–101, 1964.

[2] A. B. Owen. Empirical likelihood confidence intervals for a single functional. *Biometrika*, **75** (2):237–249, 1988.

[3] F. Hampel, C. Hennig, E. Ronchetti. A smoothing principle for the Huber and other location M-estimators. *Computational Statistics and Data Analysis*, **55** (1):324–337, 2011.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# BARTLETT CORRECTION FOR THE EMPIRICAL LIKELIHOOD METHOD FOR THE TWO–SAMPLE MEAN PROBLEM

SANDRA VUCĀNE and JĀNIS VALEINIS

*Faculty of Physics and Mathematics, University of Latvia*

Zeļļu iela 8, Rīga, LV-1002, Latvia

E-mail: sandravucane@inbox.lv, valeinis@gmx.net

The empirical likelihood method, introduced by Owen [1; 2], is the only nonparametric method that admits Bartlett adjustment. In the one–sample case Bartlett correction has been derived for the mean, for the smooth function of mean, for quantiles, for the coefficients of linear regression and also for the empirical likelihood method with nuisance parameters etc. (see [3]).

Regarding the two–sample case recently Liu et al. [4] established Bartlett correction for the two-sample mean difference, which corrected the previous result given by Jing [5]. Our initial goal was to establish Bartlett correction for the empirical likelihood in the general two-sample problems (see [6], [7]) that would include mean, distribution and quantile function differences, probability-probability, quantile-quantile plots, ROC curves and structural relationship models. Analysing the proof in [4] we found some inaccuracies. Our goal is to check whether Bartlett correction for empirical likelihood for the two-sample mean problem in [4] is correct theoretically and to end our analysis by extensive simulation study.

**REFERENCES**

[1] A. B. Owen. Empirical likelihood ratio confidence intervals for a single functional. *Biometrika*, **75** (2):237–249, 1988.

[2] A. B. Owen. Empirical likelihood ratio confidence regions. *The Annals of Statistics*, **18** (1):90–120, 1990.

[3] A. B. Owen. *Empirical Likelihood*. Chapman & Hall/CRC, 2001.

[4] Y. Liu, C. Zou and R. Zhang. Empirical likelihood for the two-sample mean problem. *Statistics & Probability Letters*, **78** (5):548–556, 2008.

[5] B. Jing. Two-sample empirical likelihood method. *Statistics & Probability Letters*, **24** (4):315–31), 1995.

[6] J. Valeinis. *Confidence bands for structural relationship models*. Phd thesis, Goerg-August-University of Goettingen, Goettingen, 2007.

[7] J. Valeinis, E. Cers and J. Cielens. Two-sample problems in statistical data modelling. *Mathematical modelling and analysis*, **15** (1):137–151, 2010.

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# ON THE FOURTH ORDER DIFFERENTIAL EGUATION OCCURING IN THE THEORY OF TRAVELING WAVES[1]

INARA YERMACHENKO[1] and FELIX SADYRBAEV[1,2]

[1] *Daugavpils University*

Parādes iela 1, Daugavpils, LV-5400, Latvia

[2] *Institute of Mathematics and Computer Science of University of Latvia*

Raiņa bulvāris 29, Rīga, LV-1459, Latvia

E-mail: `inara.jermacenko@du.lv, felix@latnet.lv`

In the theory of traveling waves for some partial differential equations the equation

$$x^{(4)} + c^2\, x'' + f(x) = 0 \tag{1}$$

appears, where $c^2$ is a parameter and $f(x)$ is $C^1$-function satisfying $f(0) = 0,\ x\, f(x) > 0$.

The homoclinic solutions of some equation are those which vanish at infinities.

In our talk we discuss the conditions under which the equation (1) has homoclinic solutions for some particular functions $f(x)$ and special choice of $c^2$.

We consider even solution of (1) under the initial conditions

$$x(0) = \alpha, \quad x'(0) = 0, \quad x''(0) = \pm\sqrt{2F(\alpha)}, \quad x'''(0) = 0, \tag{2}$$

where $F' = f$ and $F(0) = 0$. We construct functions $z(t,\, \alpha,\, T)$, which are solutions of problem (1), (2) in the interval $[-T,\, T]$ and which satisfy the linearized equation

$$y^{(4)} + c^2\, y'' + f_0\, y = 0, \qquad f_0 = \frac{df}{dx}\Big|_{x=0} \tag{3}$$

in semi-infinite intervals $(-\infty,\, -T]$, $[T,\, +\infty)$.

We use functions $z(t,\, \alpha,\, T)$ as approximations to homoclinic solutions of (1).

**REFERENCES**

[1] B. Breuer, J. Horák, P. J. McKenna, M. Plum. A computer assisted existence and multiplicity proof for traveling waves in a nonlinearly supported beam. *J. Diff. Eq.*, **224:** 60–97, 2006.

[2] L. A. Peletier, J. A. Rodríques. Homoclinic orbits to a saddle-center in a fourth order differential equation. *J. Diff. Eq.*, **203** (2):185–215, 2004.

[3] D. Smets, J. B. van den Berg. Homoclinic solutions for Swift - Hohenberg and suspension bridge type equations. *J. Diff. Eq.*, **184:** 78–96, 2002.

[4] L. A. Peletier, W. C. Troy. *Spatial Patterns Higher Order Models in Physics and Mechanics*. Birkhäuser, Boston, 2001.

[1] This work was partially supported by the Latvian Council of Science research project 09.1220

Abstracts of MMA2011, May 25–28, 2011, Sigulda, Latvia


# PARALLEL DEPTH-FIRST SIMPLICIAL PARTITION FOR COPOSITIVITY DETECTION

JULIUS ŽILINSKAS

*Vilnius University, Institute of Mathematics and Informatics*

Akademijos 4, LT-08663 Vilnius, Lithuania

E-mail: julius.zilinskas@mii.vu.lt

Copositivity plays an important role in combinatorial and quadratic optimization since setting up a linear optimization problem over the copositive cone leads to exact reformulations of combinatorial and quadratic programming problems. Optimization of linear function over copositive cone is called copositive programming [2]. Such problems are not polynomially solvable. The property is preserved when the problem is formulated as a copositive program. The complexity is moved to the cone constraint: checking whether a given matrix is copositive, is a co-NP-complete problem [3].

An algorithm for copositivity detection by simplicial partition [1] reduces the problem to that of verifying non-negativity of a quadratic form over the standard simplex and iteratively scans finer and finer simplicial partitions. Depth-first simplicial partition algorithm [4] does not require storing the whole set of simplices. This reduces memory requirements significantly and therefore enables copositivity checks of larger matrices.

In this lecture we present a parallel algorithm for copositivity detection based on depth-first simplicial partitioning. Each process runs the same algorithm, but explores different branches of the search tree depending on the number of processes, the rank of process and the level for distribution of work. We discuss numerical results on test problems showing scalability of the algorithm. The results show that parallelization enables copositivity verification of larger matrices.

## REFERENCES

[1] S. Bundfuss and M. Dür. Algorithmic copositivity detection by simplicial partition. *Linear Algebra and its Applications*, **428** (7):1511–1523, 2008. doi:10.1016/j.laa.2007.09.035

[2] M. Dür. Copositive Programming – a Survey. In: *The 14th Belgian-French-German Conference on Optimization, Leuven, Belgium, 2009*, Recent Advances in Optimization and its Applications in Engineering, M. Diehl, F. Glineur, E. Jarlebring and W. Michiels (Eds.), Springer, 2010, 3–20. doi:10.1007/978-3-642-12598-0_1

[3] K.G. Murty and S.N. Kabadi. Some NP-complete problems in quadratic and nonlinear programming. *Mathematical Programming*, **39** (2):117–129, 1987. doi:10.1007/BF02592948

[4] J. Žilinskas and M. Dür. Depth-first simplicial partition for copositivity detection, with an application to Max-Clique. *Optimization Methods and Software*, **in press** 2011. doi:10.1080/10556788.2010.544310